

Recursive Preferences, the Value of Life, and Household Finance*

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Abstract

We analyze lifecycle saving using a recursive utility model calibrated to match estimates of the value of a statistical life. The novelty of our approach is that we require preferences to be monotone with respect to first-order stochastic dominance while disentangling risk aversion and the intertemporal elasticity of substitution. We show that, with a positive value of life, risk aversion reduces each of savings, stock market participation, and annuity purchase. Risk averse agents insure against early death by consuming more when young and retaining wealth for bequests. These results contrast with those of previous studies using non-monotonic recursive models.

Keywords: lifecycle model, value of life, risk aversion, saving choices, portfolio choices, annuity puzzle, recursive utility.

JEL codes: D91, G11, J14, J17.

1 Introduction

“Nothing, they say is more certain than death, and nothing more uncertain than the time of dying” – Thomas Paine.

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The certainty of death, and the unpredictability of its timing, are fundamental features of human existence. Any analysis of an aspect of human behavior to which mortality is relevant has to reflect on how people react to lifetime uncertainty and to how they respond to opportunities to modify their mortality risk. A coherent framework for investigating the interaction between human behavior and mortality risk is particularly relevant in a period where the risk of dying could change significantly due to medical progress, or due to the occurrence of novel risks.

Although there are some exceptions, the economic literature on such matters is mostly split in two branches, depending on whether mortality risk is considered to be exogenous or endogenous. On the one hand, papers in the Household Finance literature (hereafter HF) tackle questions related to consumption, saving and financial portfolio choices over the lifecycle. Mortality is usually considered to be exogenous, and as such, the willingness to pay for mortality risk reduction tends to be not taken into consideration.

On the other hand, the literature on the value of life (hereafter VoL) considers questions related to endogenous mortality risk reduction. This literature studies the trade-off between wealth and mortality, which is key to the evaluation of public policies aiming at lowering mortality risk – such as road safety investments, public health spending or nonpharmaceutical interventions during a pandemic. Papers in this literature have paid limited attention to saving behaviors or portfolio choices and how they interact with mortality risk.

Both strands of literature were initiated with contributions relying on the same decision model: the standard additive expected utility model, as introduced by Yaari (1965) in the HF literature and used by Shepard and Zeckhauser (1984) and Rosen (1988) in the VoL literature. There was therefore a single model of rational behavior that could be used to discuss both kinds of related issues. However, the additive model was criticized by both literature strands. The HF literature emphasized the limitation of the additive model's intertwining of risk aversion and the intertemporal elasticity of substitution (IES). Some contributions to the VoL literature further found fault with the additive model for implying that agents unavoidably prefer death to life when their consumption gets small enough and when the IES is below one (see e.g., Marshall, 1984 or Rosen, 1988).

Both lines of literature tried to circumvent these difficulties by adopting recursive specifications inspired by the framework of Epstein and Zin (1989) and Weil (1990) – which are usually referred to as EZW preferences. However, the two branches of the literature followed radically different and mutually incompatible choices regarding preference parameters. Due to their focus on saving behavior and portfolio choices, papers in the HF literature typically assume a coefficient of risk aversion above

one. This, however, yields EZW specifications that are inadequate for VoL matters, since these specifications imply that people prefer to have shorter lives than longer ones (see our discussion in Section 2.3). To ensure that life is worth living in their models, papers in the VoL literature instead assume a coefficient of risk aversion below one. In this case however, unless the elasticity of substitution is restricted to be greater than one, EZW specifications are ill-defined when applied to realistic mortality patterns and yield counterfactual predictions for lifecycle behaviors (also discussed in Section 2.3).¹

Considered together, these developments in the HF and VoL literature represent a fragmented approach where a given form of rationality is used when focusing on saving behavior and portfolio choice and another one, incompatible with the former, is used when discussing endogenous changes to mortality risk. These different approaches imply contradictory associations between mortality risk and the propensity to save.² This relation between mortality risk and saving behaviors is, however, essential for understanding the impact of ongoing mortality changes on household wealth accumulation both at the micro and the macro levels, and for many issues in the economics of aging. There is therefore a need for clarification of the full role of mortality in models of household behavior, and, above all, for a consistent framework that can be used for jointly modeling choices over savings and choices related to mortality risk while being well-defined when applied to realistic survival probabilities without imposing ad-hoc restrictions on the IES or risk aversion.

The contribution of the current paper is threefold. First, we introduce a flexible, well-behaved recursive framework that can be used in both lines of literature. Central to our approach is that we restrict our attention to models which, like the standard additive expected utility model, are monotone with respect to first-order stochastic dominance. As we explain in the following paragraph, this property is essential for affording an intuitive understanding of how mortality risk and risk aversion together impact household behavior. Like models that use EZW preferences, our specification makes it possible to disentangle the IES and risk aversion, but unlike models with EZW preferences we achieve this in a framework in which preferences are monotone, and can further accommodate positive (as well

¹An exception is the model of Pashchenko and Porapakkarm (2021), who consider non-homothetic EZW preferences, allowing them to combine a positive value of life, an IES below one and a coefficient of risk aversion greater than one. However, this framework is not as tractable as the standard EZW model. See Section 5 for a lengthier discussion.

²Recursive models in the HF literature, which usually focus on the case where the IES is below one, find that the propensity to save increases with the likelihood of surviving to the next period. Falls in mortality risk would then generate an increase in (age-specific) savings rates. The opposite association is found in models from the VoL literature which assume an IES below one.

as negative) values of life, independently of the assumptions made regarding the IES or risk aversion. The novelty of this first contribution is that this is first paper to study lifecycle choices using a framework that disentangles risk aversion from IES while preserving the key characteristics of the standard additive model without imposing constraints on preference parameters (risk aversion, IES or rate of time preferences).

Second, using this framework, we formally characterize the impact of risk aversion on savings behaviors and annuity demand, in an analytically-tractable two-period framework where mortality is the only risk at play. The innovation of the second contribution is that this paper is the first to derive theoretical predictions explaining how risk aversion simultaneously impacts savings and annuity purchases. These results emphasize the role of the value of life which is shown to be fundamental for understanding the impact of risk aversion on household consumption and saving behavior in the presence of mortality risk.

Last, we provide a quantitative application of our framework to a multi-period and multi-risk setting. We compare the predictions of our model to those of the standard additive model and also contrast our results with those of previous studies that used recursive models. The starkest difference between our quantitative model and the standard additive model is in the ability of the former to rationalize low annuity demand. Low annuity demand occurs as a natural outcome of a standard calibration in our model, whereas annuity demand is counterfactually high in the standard model (a manifestation of the so-called ‘annuity puzzle’ – e.g., Yaari, 1965 or Davidoff et al., 2005). We view this as a channel which can contribute to the literature on determinedly low annuity demand, complementary to other proposed solutions to the ‘annuity puzzle’ (see Brown, 2007 for a review of this literature). We also find that the role of risk aversion differs from the one found in other studies based on recursive models, precisely because we use a well-behaved specification that features a positive value of life. The novelty of this last contribution is that our quantitative exercise is, to the best of our knowledge, the first attempt to have a flexible life-cycle model that features a wide set of choices (savings, annuity demand, portfolio choice, and bequests), while simultaneously including risks related to survival, labor income and asset return, and imposing a plausible value of life calibration. While a number of these aspects have been considered with standard additive models (see Gomes et al., 2021 for a survey) – which do not allow to analyze the role of risk aversion in isolation – we are not aware of any contribution where this is done with flexible well-behaved recursive preferences.

The keystone of our contribution is to use a model that fulfills a property of monotonicity with respect to first-order stochastic dominance. Such a property

rules out the choice of dominated strategies. Monotonicity has long been seen as a natural aspect of rationality (see for example Arrow, 1951) and it was only in the late 1980s that some non-monotone models, such as EZW preferences, became widely used in applications.³ The main purpose of EZW specifications was to allow for tractable and flexible models, which was guaranteed by preference homotheticity and the possibility of disentangling risk aversion and IES. This has facilitated the investigation of numerous research questions and has made them an important tool for the study of household behavior. The cost of the tractability and flexibility, though, is the departure from monotonicity, which is not innocuous. An agent endowed with EZW preferences can opt for some choices that offer worse outcomes in all states of the world than other available choices (see Lemma 3 in Bommier et al., 2017 or Section 2.4 of the current paper). This is analogous to following dominated strategies in a game theoretic context. Moreover, in non-monotone set-ups risk aversion no longer has a straightforward effect on agents' decisions.

To illustrate this last point, let us take a small detour, and consider an agent who faces the risk of flooding, and can purchase some flood insurance. From an ex-post point of view, if there is flooding, welfare would be improved by having purchased insurance; if there is no flooding, welfare would be hurt by having purchased it. The agent's decision about the quantity of insurance to be purchased has, however, to be taken ex-ante, before knowing whether flooding will occur. Under preference monotonicity, the decision can be seen as involving a trade-off between the welfare obtained in case of flooding and the welfare in case of no flooding. This interpretation of trading-off the welfare in different states of the world is impossible if non-monotone preferences are assumed, since the agent could take a decision that would lower the welfare in all states of the world. In a monotone framework, where choice under uncertainty amounts to trading-off welfare levels obtained in different states of the world, risk aversion simply drives how much weight is given to bad states as compared to good states. In mathematical representations, risk aversion is obtained by affording greater marginal utilities to bad states, for example through the concavity of the utility function (as in the expected utility framework) or through probability transformations (as in the dual model of Yaari, 1987). In our example, if floods are adverse events for the agents, then risk aversion would lead an agent to increase welfare in states of the world where flooding occurs, that is, risk aversion would lead them to increase insurance purchase.⁴ Note however,

³Another well-known model that is non-monotone is prospect theory, in its original formulation (Kahneman and Tversky, 1979). To remedy this non-monotonicity, Tversky and Kahneman (1986) proposed cumulative prospect theory, which is now the most commonly used version.

⁴In textbooks, this well-known relation between risk aversion and insurance demand is usually derived in the expected utility framework, but the relation extends to any model which is monotone

that in our example, an inverse relationship would hold if agent saw flooding as a positive event (for example, if flooding improved the fertility of her land).⁵ Under preference monotonicity, the relationship between risk aversion and behavior is therefore very intuitive, and naturally depends on whether the occurrence of the uncertain event is perceived as a favorable or as an unfavorable outcome.

Returning to the focus of the current paper, let us now consider a decision problem faced by an agent where the risk is death rather than flooding. An agent who cannot anticipate when she will die, and whose preferences are monotone, has to make trade-offs between the (lifetime) welfare she would obtain if dying young and that obtained if dying old. To improve welfare in the case of a short life, the agent can consume a lot when young (and thus save little) and keep resources to be bequeathed in case of death (and thus invest little in annuities). Such strategies tend to decrease welfare in case of long lives (since consumption will then be low at old ages). The effect of risk aversion on saving and annuity demand depends on whether an early death is seen as an adverse event or not, that is on the sign of the value of life. Under the plausible assumption that the value of life is positive, that is when having a short life is seen as an adverse realization, one should expect risk aversion to decrease savings and annuity purchases – in order to improve the welfare in the bad state of the world. Such intuitive reasoning is formalized in the theoretical part of the paper (Section 3), and is what drives the findings of our quantitative analysis (Section 4). This also explains why our findings differ from those in the HF literature using EZW preferences (Section 5).

Imposing preference monotonicity restricts, of course, the set of admissible preference specifications. While this rules out EZW specifications (except those where the IES is equal to one), it allows for additive and risk-sensitive preferences. The latter preferences, initially introduced by Hansen and Sargent (1995), were shown by Bommier et al. (2017) to be the only recursive preferences to afford the flexibility to separate risk aversion and intertemporal substitutability while preserving monotonicity, recursivity, and the structure of Kreps and Porteus (1978) preferences. Their recursive structure makes them amenable to computational implementation.⁶ Risk-sensitive preferences can be seen as a minimal extension of additive preferences that preserve monotonicity and recursivity but disentangle IES and risk aversion.⁷ One of the main benefit of risk-sensitive preferences is that,

with respect to stochastic dominance (see e.g., Bommier et al., 2012).

⁵If flooding is a positive event, the agent wants to “sell-short” flood insurance contracts, in other words to hold a negative amount of flooding insurance contracts. The optimum amount to be held then diminishes (i.e. becomes more negative) with agent’s risk aversion.

⁶Such preferences are used in applied settings, for instance, in Anderson (2005) or Bäuerle and Jaśkiewicz (2018).

⁷It would be possible to consider other preferences by relaxing recursivity. Recursivity

relying on preference monotonicity, they yield an intuitive interpretation of the impact of risk aversion on household decisions in the presence of mortality risk.

This paper proceeds as follows. In Section 2, we provide a theoretical review of recursive preferences, including additive, EZW and risk-sensitive preferences. We highlight, in particular, the meaning of preference monotonicity. In Section 3, we consider a simple two-period model, in which we derive theoretical results regarding the impact of risk aversion on saving and annuity demand when the only risk is mortality risk. In Section 4, we outline our quantitative model with multiple periods and multiple risks and discuss its implications for saving behavior and, in particular, annuity demand. In Section 5, we relate our results on the role of risk aversion to previous findings in the HF and VoL literature. Section 6 concludes.

2 Recursive models

2.1 The additive model

The most popular framework in both the HF and the VoL literature is the time-additive expected utility model. To link with the rest of the paper, we provide its recursive definition:

$$U_t = (1 - \beta)u(c_t) + \beta E[U_{t+1}], \quad (1)$$

where U_t is utility at time t , $\beta \in (0, 1)$ is a time preference parameter and $u(c_t)$ is the instantaneous utility derived from consumption at time t .⁸ Accounting for mortality is achieved by assigning a utility level, u_d , obtained when death occurs. We follow the VSL literature and assume that this level of utility, u_d , is independent of the age of death (Shepard and Zeckhauser, 1984; Rosen, 1988, among many others).

As it is generally assumed that mortality risk is independent of other risks, the expectation can be decomposed in two stages, one that accounts for the risk of mortality and one that accounts for other risks. Formally denoting the utility conditional on being alive at time t by V_t and the probability of surviving from period t to $t + 1$ by π_t the recursion (1) yields:

$$V_t = (1 - \beta)u(c_t) + \beta (\pi_t E[V_{t+1}] + (1 - \pi_t)u_d). \quad (2)$$

Additive preferences, as defined in the above equation, are invariant when changing

mostly serves tractability and allows us to use standard resolution techniques, such as dynamic programming.

⁸The additive model is frequently defined using the recursion $U_t = u(c_t) + \beta E[U_{t+1}]$, which is, of course, equivalent to (1), up to a multiplicative renormalization of the function u .

u and u_d (and V_t) by the same positive affine transformation. It is therefore possible to assume, without loss of generality, that the utility of death is set to zero ($u_d = 0$). If one assumes a constant IES, the function u has to be of the form $u(c) = u_l + K \frac{c^{1-\sigma}}{1-\sigma}$ for some constant u_l (the subscript l stands for “life”) and a positive scalar K . The IES is then given by $\frac{1}{\sigma}$. By (multiplicative) normalization, the scalar K can be set equal to 1. The parameter u_l , which determines the utility gap between life and death, is thus an important preference parameter. Setting u_l to zero would not be a mere normalization but would imply making a non-trivial assumption on the value of life. It is noteworthy that when mortality is exogenous, the constant u_l only contributes to an exogenous additive term that has no impact on the ordering of consumption profiles. This explains why the constant u_l is generally ignored in all studies that assume an exogenous mortality pattern but is explicitly mentioned in the VoL literature (see, for example, the discussion in Hall and Jones, 2007).

The additive specification has been criticized for its lack of flexibility. In line with Epstein and Zin (1989) and Weil (1990), papers in the literature expressed concerns regarding its inability to disentangle the IES from risk aversion.⁹

2.2 Recursive models with mortality

The search for greater flexibility led researchers contributing to each of the HF and the VoL literatures to adopt non-additive recursive models for preferences. These models assume that utility U_t at any date t is defined by the following recursion:

$$U_t = f^{-1}((1 - \beta)u(c_t) + \beta\phi^{-1}(E[\phi f(U_{t+1})])), \quad (3)$$

where ϕ is an increasing function representing risk preferences and f is a normalization device that can be any increasing function. As in the additive model of Section 2.1, $\beta \in (0, 1)$ is a time preference parameter and $u(c_t)$ is the instantaneous utility function. The function f has no impact on preferences and its role is to facilitate convenient representations of the recursion. A common choice is $f(x) = x$, as in the additive specification (1), and which we will use for the representation of risk-sensitive preferences in Section 2.5. Another common choice for f , most often introduced in the case of EZW preferences – see Section 2.3 – involves using the same CRRA function as that used for the instantaneous utility:

⁹In addition, some papers in the VoL literature emphasized the fact that when the IES is smaller than one (i.e., $\frac{1}{\sigma} < 1$), life becomes unavoidably worse than death when consumption gets small enough (see, e.g., Rosen, 1988). This creates an incentive to enter into Russian-roulette games, so as to avoid having to live in a state which is worse than death. This feature may however be seen as a theoretical curiosity, with no consequences in applications as long as consumption stays above the threshold that would make life just as good as death.

$$f(x) = u(x) = \frac{x^{1-\sigma}}{1-\sigma}.^{10}$$

As in the additive case, we can derive, from equation (3), the recursion defining the utility conditional on being alive at time t :

$$V_t = f^{-1} \left((1 - \beta)u(c_t) + \beta\phi^{-1} (\pi_t E[\phi f(V_{t+1})] + (1 - \pi_t)\phi f(u_d)) \right). \quad (4)$$

Here, again, u_d denotes the constant utility level assigned to death and mortality risk is assumed to be independent of other risks.

2.3 Homothetic EZW preferences

A popular recursive specification is that of Epstein and Zin (1989) and Weil (1990), which is obtained from equation (3), with $u(c) = f(c) = \frac{c^{1-\sigma}}{1-\sigma}$, and $\phi(x) = \frac{1}{1-\gamma} ((1 - \sigma)x)^{\frac{1-\gamma}{1-\sigma}}$. This yields:

$$U_t = \left((1 - \beta)c_t^{1-\sigma} + \beta \left(E[U_{t+1}^{1-\gamma}] \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}}. \quad (5)$$

When $\sigma = \gamma$, we obtain the additive model – up to the transformation by f . With $\gamma > \sigma$, as is usually assumed in the HF literature, EZW exhibit preference for early resolution of uncertainty. The magnitude of the implied timing premia is discussed in Epstein et al. (2014).

The recursion (4), which gives the utility conditional on being alive, is now:

$$V_t = \left((1 - \beta)c_t^{1-\sigma} + \beta \left(\pi_t E[V_{t+1}^{1-\gamma}] + (1 - \pi_t)u_d^{1-\gamma} \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}}. \quad (6)$$

It is worth emphasizing that in specification (6), choosing a specific value for u_d is not a normalization choice, since changing u_d would impact individual preferences. Since γ is an arbitrary real number (with $\gamma > 0$ and $\gamma \neq 1$), $x \mapsto x^{1-\gamma}$ is not defined for negative values of x . Thus, for the recursion (6) to be well-defined, the constant u_d cannot be negative.

The specification (6) is not homothetic (multiplying all consumption levels by a scalar affects utilities in a non-trivial way), unless one sets $u_d = 0$ or $u_d = \infty$ (depending on the sign of $1-\gamma$) so that $u_d^{1-\gamma} = 0$. Most papers, with Pashchenko and Porapakarm (2021) as a noteworthy exception, focus on homothetic specifications. The HF and VoL literatures at this point follow two different (and incompatible) routes. In the HF literature, the standard assumption is to choose a coefficient of risk aversion $\gamma > 1$. In order to get a homothetic specification, it is assumed that $u_d^{1-\gamma} = 0$ and thus, implicitly, that the utility of death is $u_d = +\infty$. The recursion

¹⁰Such a normalization implies that an infinitely long consumption path providing the same consumption c in all periods yields a lifetime utility equal to c .

(6) reduces then to:

$$V_t = \left((1 - \beta)c_t^{1-\sigma} + \beta\pi_t^{\frac{1-\sigma}{1-\gamma}} \left(E[V_{t+1}^{1-\gamma}] \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}}, \quad (7)$$

as is the case, for example, in Gomes and Michaelides (2008). Clearly, setting $u_d = +\infty$ involves assuming that death is preferable to life, regardless of consumption levels. This has the consequence that utility is declining with survival probability ($\frac{\partial V_t}{\partial \pi_t} < 0$).¹¹ Since the HF literature focuses on cases where mortality is exogenous, one could think that this assumption is harmless. We will show in Section 3 that this is not the case. We also note in Section 5 that this assumption of a negative value of life explains the difference between the findings in this line of literature and ours.

Assuming that death is preferable to life precludes the use of the model to study VoL issues. For this reason, rather than assuming $\gamma > 1$ and $u_d = +\infty$, Hugonnier et al. (2013) and Córdoba and Ripoll (2017) suggest the imposition of $\gamma < 1$ and $u_d = 0$, which again yields the recursion (7). An additional issue arises, however, when considering the case where the IES is smaller than one ($\frac{1}{\sigma} < 1$), which is the relevant case in most of the literature (see the meta-analysis of Havránek, 2015). When applied to realistic mortality patterns (with π_t small at old ages), the recursion (7) admits a unique solution, which is the null function: $V_t = 0$ independent of the consumption profile. Therefore, all feasible consumption profiles are optimal. Moreover, computing first-order conditions implied by recursion (7), while ignoring that the solution is $V_t = 0$, yields consumption profiles where agents consume almost nothing early in the lifecycle to sustain a very high consumption at old ages. Such models predict an age-dependent discount rate of $\frac{1}{\beta\pi_t^{\frac{1-\sigma}{1-\gamma}}} - 1$ (instead of $\frac{1}{\beta\pi_t} - 1$ for standard additive models), which becomes negative, and even converges to -1 , when survival probabilities get small. We will illustrate this phenomenon in the context of a simple two-period savings example in Section 2.4. These implications are avoided in Hugonnier et al. (2013) by assuming a model of perpetual youth, where mortality risk remains small at all ages. In Córdoba and Ripoll (2017), these issues are avoided by the introduction of an ad hoc (exogenous) health profile, which has the property that greater health implies a lower utility level. The problem of a negative value of life is solved by introducing another one:

¹¹Equation (7) yields $\frac{\partial V_t}{\partial \pi_t} = \frac{\beta}{1-\gamma} \left(E[V_{t+1}^{1-\gamma}] \right)^{\frac{1-\sigma}{1-\gamma}} V_t^\sigma$, which has the same sign as $1 - \gamma$. See Section 5 for further discussion.

that of a negative value of health.¹² We refer to Bommier et al. (2021) for an extensive discussion of these issues. Zhang et al. (2018) provide further criticisms regarding the role of the σ parameter when using recursion (7).

We conclude that there is room for a more general approach, which can allow risk aversion and the elasticity of intertemporal substitution to be disentangled without placing constraints on the signs or values of those parameters or on the value of life. Rather than bringing a new specification chosen for its tractability, we suggest the imposition of a natural property: preference monotonicity.

2.4 Preference monotonicity

Monotonicity with respect to first-order stochastic dominance (see Bommier and LeGrand, 2014a for a formal definition) stipulates that an agent will not take an action if another one would be preferable in all future circumstances. This property, which is similar to the elimination of dominated strategies in game theory, seems to be a reasonable requirement when modeling individual rationality (see the discussion in Arrow, 1951). The standard additive model is monotone, but only few recursive extensions are monotone. Bommier et al. (2017) show that EZW preferences are non-monotone, except when an IES equal to one is assumed.¹³

The non-monotonicity feature of EZW preferences can be illustrated in various settings. For example, Bommier et al. (2017) consider the case of saving behavior with uncertain income and asset returns, in the absence of mortality risk. To get closer to the problem considered by the current paper, we give below an example where EZW preferences yield dominated choices in settings where the uncertainty is solely related to mortality.

The example of a two-period savings problem. We consider a two-period saving problem of an agent who has EZW preferences and who is faced with an exogenous probability ($\pi_0 \in (0, 1)$) of surviving to the second period. The agent is

¹²Córdoba and Ripoll (2017) extend (7) by assuming that:

$$V_t = \left((1 - \beta)H_t c_t^{1-\sigma} + \beta \pi_t^{\frac{1-\sigma}{1-\gamma}} (E[V_{t+1}^{1-\gamma}])^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}},$$

where H_t is an exogenous health profile. By choosing an appropriate health profile, it is then possible to match any possible consumption profile. The calibration in their paper leads to the choice of a profile H_t that rapidly declines with age – so as to mechanically counterbalance the counterfactual increase in consumption arising in that model when the IES is smaller than one. When the IES is smaller than one (i.e., $\frac{1}{\sigma} < 1$), the model implies that health has a negative impact on welfare: $\frac{\partial V_t}{\partial H_t} < 0$.

¹³When the IES equals one (i.e., the limit case where $\sigma \rightarrow 1$), EZW preferences enter in the class of risk-sensitive preferences that we consider in Section 2.5.

endowed with an initial amount of wealth w_0 , and no additional source of revenue, has to decide in period 0, how much to save for period 1. We assume that the agent has to choose an amount b to be invested in a risk-free bond whose return is equal to $\frac{1}{\beta}$. Using equation (7), the decision problem is therefore:

$$\max_{b \in \mathbb{R}_+} \left((1 - \beta)(w_0 - b)^{1-\sigma} + \beta \pi_0^{\frac{1-\sigma}{1-\gamma}} (1 - \beta) \left(\frac{b}{\beta}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \quad (8)$$

Ex ante, the agent foresees two possible states of the world. Either she lives only one period, and (ex-post) lifetime utility would be maximized by consuming everything in period 0 (zero savings: $b = 0$). Or she lives two periods and (ex-post) lifetime utility would be maximized with $b = \frac{w_0}{1+\beta^{-1}}$ in order to have the same consumption level in both periods. The agent however, has to make a saving decision before knowing which state will be realized. Choosing a saving level above $\frac{w_0}{1+\beta^{-1}}$ would be a dominated choice since saving exactly $\frac{w_0}{1+\beta^{-1}}$ would provide a higher ex-post utility whether she survives or not.

Simple derivations show that the solution to (8) is $b^* = \frac{w_0}{1+\beta^{-1} \pi_0^{\frac{1}{\sigma} \frac{\sigma-1}{1-\gamma}}}$. The interesting case corresponds to $\frac{\sigma-1}{1-\gamma} > 0$, which occurs when risk aversion, γ , and the IES, $\frac{1}{\sigma}$, are both either below or above 1. In such a case, we obtain that $b^* > \frac{w_0}{1+\beta^{-1}}$, indicating that the agent opts for a dominated strategy. When π_0 becomes very small, then b^* even gets close to w_0 . In other words, when the agent is almost sure of dying at the end of the period 0, she decides to consume almost nothing in that period, to keep all her resources for period 1 – a period that she will almost surely never see. Such unpleasant features extend to multi-period settings, with similar conclusions for saving behaviors.

In our example, non-monotonic choices appear when $\frac{\sigma-1}{1-\gamma} > 0$, but as shown in Bommier et al. (2017), the non-monotonicity of EZW preferences is generic, except when an IES of one is assumed. In such cases, EZW preferences overlap with risk-sensitive preferences that we introduce below.

2.5 Risk-sensitive preferences

Imposing monotonicity does not preclude the use of non-additive recursive models. It does, however, dramatically reduce the set of recursive specifications that can be used. It was shown in Bommier et al. (2017) that the only class of Kreps and Porteus (1978) recursive preferences that is monotone and flexible enough to disentangle risk aversion from the IES is the one provided by risk-sensitive

preferences. Such preferences correspond to the recursive setting where:

$$U_t = (1 - \beta)u(c_t) - \frac{\beta}{k} \log \left(E_t[e^{-kU_{t+1}}] \right). \quad (9)$$

As was the case for EZW preferences, recursion (9) is a particular case of equation (3), where $f(x) = x$, and $\phi(x) = \frac{1-e^{-kx}}{k}$. The parameter k governs risk aversion, where larger values of k are associated with more risk averse behavior. When $k \rightarrow 0$, recursion (9) converges toward the standard additive model of Section 2.1. Risk-sensitive preferences with $k > 0$ exhibit a preference for early resolution of uncertainty, the intensity of which vanishes when β gets close to one (Bommier et al., 2017). The limit case $\beta = 1$ corresponds to the multiplicative model of Bommier (2013) and Bommier and LeGrand (2014b) that rules out pure time preferences.¹⁴

Intuitively, preferences represented by recursion (9) are monotone, as the term $(1 - \beta)u(c_t)$ can be included inside the expectation. That is, recursion (9) can also be written as $U_t = -\frac{\beta}{k} \log \left(E[e^{-\frac{k}{\beta}((1-\beta)u(c_t) + \beta U_{t+1})}] \right)$, implying that the choices at time t involve maximizing an expectation, just like in the expected utility framework.

A feature that is worth emphasizing is that preferences defined by (9) are invariant when adding a constant to the instantaneous utility function u . The risk-sensitive model thus preserves one of the invariance properties of the additive specification. This is convenient for normalization matters since, just like in the additive model, it can be assumed that $u_d = 0$ with no generality loss. The recursion (4), giving utility conditional on being alive then becomes:

$$V_t = (1 - \beta)u(c_t) - \frac{\beta}{k} \log \left(\pi_t E[e^{-kV_{t+1}}] + 1 - \pi_t \right).$$

Interestingly risk-sensitive preferences do not constrain the IES to be constant, since the function u can be arbitrarily general.

3 Risk aversion, the value of live and saving behavior

As discussed in the introduction, with monotone preferences, a utility maximizing choice under uncertainty can be seen as involving trade-offs between the ex-post utilities obtained in all states of the world. Further, increasing risk aversion can be understood as putting greater weight on “bad states” of the world. This section

¹⁴To be fully precise, the multiplicative model is obtained by first re-normalizing the instantaneous utility function, setting $v(c_t) = (1 - \beta)u(c_t)$ and then taking the limit $\beta \rightarrow 1$ while keeping the function v unchanged.

formalizes this view of the role of risk aversion and also emphasizes the importance of correctly distinguishing between good and bad states of the world. Regardless of whether mortality risk is endogenous or exogenous, risk aversion will have opposite effects depending on whether a long life is assumed to be a good outcome (i.e., the value of mortality risk reduction is positive) or a bad outcome (i.e., the value of mortality risk reduction is negative).

Formally, we consider here an agent who lives for at most two periods. In period 0, the agent is endowed with a level of wealth w_0 and has to make saving and annuity purchase decisions. More precisely, the agent may invest in bonds, which yield a safe return of R^f and which are bequeathed in case of death, or in annuities, with return $\frac{R^f}{\pi_0}$, but which are not bequeathable. Denoting by c_0 and c_1 the consumption in the first and second periods, by b the amount invested in bonds, by a the amount invested in annuities, and by x the amount bequeathed in case of death, we have: $w_0 = c_0 + b + a$, $c_1 = R^f(b + \frac{a}{\pi_0})$, and $x = R^f b$. The agent is endowed with risk-sensitive preferences, as represented by equation (9). However, the utility associated with death is not constant and equal to u_d anymore, but depends on the size of the bequest left by the agent. We denote by $v(x)$ the utility associated with bequeathing the amount x . The recursion (4) defining the utility V_0 representing the agent's preferences becomes, in the presence of bequests:

$$V_0 = (1 - \beta)u(c_0) - \frac{\beta}{k} \log \left(\pi_0 e^{-k(1-\beta)u(c_1)} + (1 - \pi_0) e^{-k(1-\beta)v(x)} \right). \quad (10)$$

We further assume here that $u(c) = u_l + \frac{c^{1-\sigma}}{1-\sigma}$ and $v(x) = u_d + \theta \frac{x^{1-\sigma}}{1-\sigma}$, where the scalar $\theta \geq 0$ quantifies the strength of the bequest motives. There is no consensus on the formulation of v (known as a 'warm-glow' bequest function), but the form we have chosen is the same as in Cocco et al. (2005), Inkmann et al. (2011) and Yogo (2016).¹⁵ With no loss of generality, we can normalize the utility with $u_d = 0$. Note that further constraining utility by setting, for instance, $u_l = 0$ would not be a mere normalization and would impose constraints on the value of mortality risk reduction. The larger is u_l , the larger is the utility gap between life and death and the larger is the value of mortality risk reduction. In particular, it follows from (10) that:

$$\frac{\partial V_0}{\partial \pi_0} = \frac{\beta}{\pi_0 e^{-k(1-\beta)u(c_1)} + (1 - \pi_0) e^{-k(1-\beta)v(x)}} \frac{e^{-k(1-\beta)(u_l + \frac{c_1^{1-\sigma}}{1-\sigma})} - e^{-k(1-\beta)\theta \frac{x^{1-\sigma}}{1-\sigma}}}{-k}, \quad (11)$$

¹⁵Other papers, such as De Nardi (2004), Lockwood (2012) or Bommier and LeGrand (2014b) consider a bequest utility of the form $\theta \frac{(\bar{x}+x)^{1-\sigma}}{1-\sigma}$ that is not homothetic but enables bequests to be modeled as a luxury good. We will use such a specification in the calibrated multi-period quantitative model presented in Section 4.

implying that, for given c_1 and w , people prefer longer lives ($\frac{\partial V_0}{\partial \pi_0} > 0$) if u_l is above $\frac{\theta x^{1-\sigma} - c_1^{1-\sigma}}{1-\sigma}$ while people prefer shorter lives ($\frac{\partial V_0}{\partial \pi_0} < 0$) if u_l is below that threshold.

Let b_k and a_k be the optimal saving and annuity choices of an agent with risk aversion k . Formally, the consumption-saving program can be written as:

$$(b_k, a_k) = \arg \max_{(b,a) \in \mathbb{R}_+^2} (1 - \beta)u(w_0 - b - a) - \frac{\beta}{k} \log \left(\pi_0 e^{-k(1-\beta)u(R^f b + R^f \frac{a}{\pi_0})} + (1 - \pi_0) e^{-k(1-\beta)v(R^f b)} \right) \quad (12)$$

We denote by $c_{0,k} = w_0 - b_k - a_k$ and $c_{1,k} = R^f(b_k + \frac{a_k}{\pi_0})$ the corresponding optimal first- and second-period consumption levels. Here again, the case of additively separable preferences is obtained by taking the limit as $k \rightarrow 0$.

Proposition 1 *Consider the consumption-saving problem in equation (12).*

If $k = 0$, then the choices a_0 and b_0 , and hence the consumption levels $c_{0,0}$ and $c_{1,0}$, are independent of u_l .

If $k > 0$ and $a_k > 0$, we have:

- if u_l is such that $\frac{\partial V_0}{\partial \pi_0} > 0$ at the optimum (i.e., if the value of mortality risk reduction is positive), then $\frac{\partial a_k}{\partial k} < 0$, $\frac{\partial b_k}{\partial k} > 0$, $\frac{\partial c_{0,k}}{\partial k} > 0$ and $\frac{\partial c_{1,k}}{\partial k} < 0$;*
- if u_l is such that $\frac{\partial V_0}{\partial \pi_0} < 0$ at the optimum (i.e., if the value of mortality risk reduction is negative), then $\frac{\partial a_k}{\partial k} > 0$, $\frac{\partial b_k}{\partial k} < 0$, $\frac{\partial c_{0,k}}{\partial k} < 0$ and $\frac{\partial c_{1,k}}{\partial k} > 0$.*

The proof can be found in Appendix A.1. To the best of our knowledge, Proposition 1 is the first result that characterizes the role of risk aversion simultaneously for riskless savings and annuity investment and that emphasizes the key role of the sign of the value of life in these relationships.

Here we will comment on the results of Proposition 1, focusing on the case where $k > 0$ and $a_k > 0$ (which avoids corner solutions). We begin with the case where $\frac{\partial V_0}{\partial \pi_0} > 0$ at the optimum, that is, when the agent would prefer to have a greater survival probability. We find that increasing risk aversion decreases second-period consumption ($\frac{\partial c_{1,k}}{\partial k} < 0$), implying therefore that overall savings payoffs are reduced ($\frac{\partial}{\partial k}(b_k + \frac{a_k}{\pi_0}) < 0$). Moreover, savings shift towards safe assets ($\frac{\partial b_k}{\partial k} > 0$) at the expense of annuity purchases ($\frac{\partial a_k}{\partial k} < 0$). In fact, the greater the risk aversion, the more the agent is concerned about the bad state, that is, the case where she would die after the first period. To increase lifetime utility derived in the case that the bad state is realized, she increases her first period consumption ($\frac{\partial c_{0,k}}{\partial k} > 0$) and the amount she leaves as bequest ($\frac{\partial b_k}{\partial k} > 0$). At the same time, she purchases a smaller amount of annuities. These choices make a short life a less adverse outcome. Of

course, this comes at the cost of a lower second-period consumption ($\frac{\partial c_{1,k}}{\partial k} < 0$) and of a lower lifetime utility in case of survival (we prove in Appendix A.1 that $\frac{\partial}{\partial k}(u(c_{0,k}) + \beta u(c_{1,k})) < 0$).

Interestingly, the findings look very different in the (counterfactual) case where $\frac{\partial V_0}{\partial \pi_0} < 0$, that is, when the agent would prefer to have a lower survival probability. As shown in Proposition 1, the impact of an increase in risk aversion is systematically reversed compared to the case where the value of mortality risk reduction is positive. This follows intuitively from the fact that the impact of an increase in risk aversion is to put greater weight on the bad state, and thus mechanically depends on which state is actually the worst. When $\frac{\partial V_0}{\partial \pi_0} < 0$, the bad state is the one where the agent lives for two periods, and an agent with $k > 0$ purchases a larger amount of annuities than an agent with additive preferences ($k = 0$), precisely to make such a “bad outcome” not so bad. As we will see in Section 5, such matters related to the sign of the value of life are key to understanding how our results contrast with those of previous contributions.

4 A quantitative lifecycle model

Having argued why the value of life matters even in models with exogenous mortality risk, this section outlines a multi-risk, multi-period, quantitative lifecycle model which can be used to study the interplay between that risk, saving behavior, portfolio choice and annuity purchases. The model’s innovation is to ensure, by using risk-sensitive preferences, preference recursivity and monotonicity while matching empirical estimates of the value of mortality risk reduction. The model structure is very rich. It features risks that affect: (i) mortality, (ii) income, and (iii) asset returns. Agents also face a complex portfolio choice that involves a riskless asset, a risky asset, and annuity holdings.

4.1 The setup

We consider an economy of agents endowed with risk-sensitive preferences who face risks over mortality, income and asset returns. Agents may save through a bond, a risky asset and may insure against longevity risk by purchasing an annuity. Time is discrete, a model period is a year, and time t corresponds to biological age minus 20. Agents enter the model at the start of working life, at $t = 0$. There is a single consumption good, whose price serves as a numeraire.

Mortality risk. Agents face mortality risk, which is assumed to be exogenous and independent of all other risks. If alive at date t , agents survive to date $t + 1$ with probability π_t . There exists a date T_M , such that the probability of living after T_M is $\pi_{T_M} = 0$.

Labor income risk. At any age, when alive, agents receive an income denoted y_t . They exogenously retire at date T_R . During retirement ($t \geq T_R$), agents receive an annual pension income $y_t = y^R$. During working life ($t < T_R$), agents earn a risky labor income $y_t = y_t^L$, defined by $\ln y_t^L = \mu_t + \zeta_t$. The sequence $(\mu_t)_{t \geq 0}$ is a deterministic process that depends on age (see Appendix C.1 for further details), and $(\zeta_t)_{t \geq 0}$ is an AR(1) stochastic component, with persistence parameter ρ and innovation $(\nu_t)_{t \geq 0}$, which is IID normally distributed with mean 0 and variance σ_ν^2 . We denote average earnings over working life as \bar{y} .

Financial risk and security markets. Agents can save through bonds and stocks and can purchase an annuity. The bond pays a constant risk-free gross return, R^f . The stock yields a risky return, defined as: $R_t^s = R^f + \omega + \nu_t$, where ω represents the average risk premium of stocks over bonds, while the stochastic component of the risky return $(\nu_t)_{t \geq 0}$ is an IID normally distributed process with mean 0 and variance σ_ν^2 .

Agents must pay a cost $F \geq 0$ to participate in the stock market, which may be interpreted as the opportunity cost of discovering how the stock market works. In our baseline model, we assume it is a flat once-in-a-lifetime cost: if the cost is paid at a given date t by an agent, they can freely trade stocks at date t and at any date afterwards.¹⁶

Finally, an annuity can be purchased in the period before retirement ($T_R - 1$). The annuity is a financial asset that pays one unit of income every period from T_R , as long as its holder is alive. The price of a single unit of annuity income, q , is:

$$q = (1 + \delta) \sum_{\tau=1}^{T_M - T_R} \frac{\prod_{s=0}^{\tau} \pi_{T_R - 1 + s}}{(R^f)^\tau}, \quad (13)$$

where the parameter $\delta \geq 0$ is a loading factor on annuity. When $\delta = 0$, the annuity is actuarially fair and its price equals the discounted present value of future payoffs. The larger is δ , the further is annuity pricing from actuarial fairness. There is one final annuity market imperfection. Following Pashchenko (2013), annuity purchases below a minimum threshold ($\underline{a} > 0$) are not allowed.

¹⁶In Appendix D.3, we investigate another participation structure, where the cost must be paid in every period that an agent wants to buy stocks.

Choices and constraints. If an agent is alive, her resources at the beginning of the period consist of her wealth, comprising bond, annuity and stock payoffs plus labor income earned, or public pension income received, in the period. Resources are used for consumption as well as the purchase of bonds, annuities, and stocks. The budget constraint of a living agent at date t can then be expressed as follows:

$$c_t + qa_t + b_t + s_t + \mathbb{1}_{\eta_t=1}\mathbb{1}_{\eta_{t-1}=0}F = y_t + w_t, \quad (14)$$

$$\text{with: } w_t = a_{T_R-1}\mathbb{1}_{t \geq T_R} + R^f b_{t-1} + R_t^s s_{t-1}, \quad (15)$$

where c_t and w_t are consumption and wealth in period t and b_t , s_t and a_t are, respectively, the quantity of bonds, stocks and annuities purchased in period t . The index η_t reflects market participation status and is equal to 0 if she has never paid the participation cost before and therefore never held stocks. The term $\mathbb{1}_{\eta_t=1}\mathbb{1}_{\eta_{t-1}=0}F$ in equation (14) represents the fixed once-in-a-lifetime cost of participation. Annuity income is received from age T_R and is therefore equal to $a_{T_R-1}\mathbb{1}_{t \geq T_R}$. No asset, including annuities, can be sold short. These constraints are summarized here:

$$s_t = 0 \text{ if } \eta_t = 0, \quad (16)$$

$$a_t = 0 \text{ if } t \neq T_R - 1, \quad (17)$$

$$a_{T_R-1} = 0 \text{ or } a_{T_R-1} \geq \underline{a}, \quad (18)$$

$$b_t \geq 0, s_t \geq 0 \text{ and } c_t > 0 \quad (19)$$

If an agent is dead at date t , she bequeaths bonds and stocks, but not annuities. The bequest x_t amounts to:

$$x_t = R^f b_{t-1} + R_t^s s_{t-1}. \quad (20)$$

A feasible allocation is a sequence of choices $(c_t, b_t, a_t, s_t, x_t, \eta_t)_{t \geq 0}$ satisfying the constraints (14)–(20). The set of feasible allocations is denoted \mathcal{A} .

4.2 Preferences and agents' program

Intertemporal preferences. Agents have risk-sensitive preferences. The utility of an agent at age t when alive, V_t , is defined through the following recursion:

$$V_t = (1 - \beta)u(c_t) - \frac{\beta}{k} \log \left(\pi_t E_t \left[e^{-kV_{t+1}} \right] + (1 - \pi_t) E_t \left[e^{-k(1-\beta)v(x_{t+1})} \right] \right), \quad (21)$$

where $v(x_t)$ is the instantaneous utility obtained upon dying and bequeathing x_t . Utility is normalized here so that being dead while leaving no bequest provides

utility $v(0) = 0$.¹⁷

Instantaneous utility function specification. We assume that agents have a constant IES. Formally,

$$u(c) = \begin{cases} u_l + \frac{c^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1, \\ u_l + \log(c) & \text{if } \sigma = 1, \end{cases} \quad (22)$$

where $\sigma > 0$ is the inverse of the IES, and u_l is a parameter that provides the instantaneous utility derived when alive and consuming one unit of consumption ($u(1) = u_l$). It can also be interpreted as the difference in utility between being alive and consuming one unit and being dead and bequeathing nothing ($u_l = u(1) - v(0)$). Since utility has already been normalized when assuming $v(0) = 0$, we cannot additionally set u_l to an arbitrary value. This parameter must, therefore, be carefully calibrated. Note that when $\sigma > 1$, there necessarily exists a threshold \underline{c} below which $u(c) < 0$, implying that agents would prefer to die and leave no bequest rather than staying alive. In theory, this threshold could be used to calibrate u_l . However, this would involve basing the calibration on extreme cases (suicides), for which our model is surely ill-suited (a suicidal decision is a complex multi-dimensional decision that, needless to say, involves more than poverty). A preferable calibration strategy, which we will use in Section 4.4, relies on using agents' decisions that relate to safety. Since a higher value of u_l means a higher valuation of being alive relative to being dead, the value of u_l should be reflected in the financial decisions that agents make to lower their mortality risks, for example when investing in safer (but more expensive) cars or opting for safer (but lower paid) jobs.

The utility derived from bequests, $v(x)$, is assumed to be continuous, increasing in the bequest amount, and to exhibit bounded and decreasing marginal utility. The functional form we use has been widely applied (see e.g., De Nardi, 2004, De Nardi et al., 2010, Ameriks et al., 2011, and Lockwood, 2012 and 2018). Formally,

$$v(x) = \begin{cases} \frac{\theta}{1-\sigma} [(\bar{x} + x)^{1-\sigma} - \bar{x}^{1-\sigma}] & \text{if } \sigma \neq 1, \\ \theta \log\left(\frac{\bar{x}+x}{\bar{x}}\right) & \text{if } \sigma = 1, \end{cases} \quad (23)$$

where σ is the inverse of the IES used in the expression (22) defining the function u , while $\theta \geq 0$ governs the strength of the bequest motive. With $\bar{x} > 0$, bequests are a luxury good, as has been shown by, for example, Hurd and Smith (2002).

¹⁷With such a normalization the continuation utility when dead, $(1 - \beta)v(x_{t+1}) + \beta v(0)$, simplifies to $(1 - \beta)v(x_{t+1})$.

The derivative $v'(0)$ is finite, so that agents bequeath only when their wealth is large enough (an empirical regularity documented by e.g., De Nardi, 2004).

Agents’ program. The agents’ problem involves determining the feasible allocation in the set \mathcal{A} that maximizes utility defined in (21). There is no analytical solution to this problem. We therefore solve the model numerically. In short, state variables are discretized and decision rules are obtained for points on the grid by backwards induction from the last period. Linear approximation is used to evaluate the value function at points off the grid and integration over earnings and asset price shocks is carried out using Tauchen (1986). Further details are given in Appendix B.

4.3 Value of mortality risk reduction

To properly calibrate u_l , we will consider the marginal rate of substitution between survival probability and wealth, which quantifies how much a given agent is willing to pay – in terms of wealth – to reduce her mortality risk. This marginal rate of substitution is most often called the *value of a statistical life* (VSL, henceforth), though there have been recent recommendations to use the terminology “value of mortality risk reduction”.¹⁸ The VSL at date t , denoted VSL_t , is defined as:

$$VSL_t = \frac{\frac{\partial V_t}{\partial \pi_t}}{\frac{\partial V_t}{\partial w_t}}, \quad (24)$$

where w_t is wealth and is given in equation (15). This definition is standard and is used in Rosen (1988), for instance. The formal expressions for the risk-sensitive and additive models – that will be used to calibrate u_l – can be found in Appendix A.2.

Empirical literature on the value of a statistical life. The value of mortality risk reduction is a central parameter for cost-benefit analyses in many policy realms. This includes evaluating environmental policy (see U.S. Environmental Protection Agency 2011 where the value of mortality risk reduction is central in estimating the benefits of the Clean Air Act), transport policy (see US Department of Transportation, 2016 on quantifying the benefit of road safety rules) and health policy (Murphy and Topel, 2006 and Hall and Jones, 2007 or also Greenstone and Nigam, 2020, Hall et al. 2020, Hammitt, 2020, or Robinson et al., 2020 in the context of the Covid-19 pandemic).

¹⁸See <https://www.epa.gov/environmental-economics/mortality-risk-valuation> for a discussion.

There are two distinct approaches that have been used to estimate the value of mortality risk reduction. The first is a revealed preference approach which estimates it from observed decisions by individuals (e.g., from compensating differentials associated with risky jobs or willingness to pay for safety features on vehicle purchases). The second is a stated preference approach, where individuals' valuations are explicitly elicited by a survey. Both approaches provide a relatively broad range of estimates. This is not surprising, as any estimate of willingness-to-pay for mortality risk reduction will depend on individual preferences and individual financial and demographic characteristics. Broad overviews of the literature and details on the range of estimates that have been reported can be found in Viscusi and Aldy (2003) and Kniesner and Viscusi (2019).

4.4 Calibration

Demographics, endowments and asset market parameters. We describe here how externally-set parameters, relating to demographics, endowments and asset markets, are chosen. Table 1 provides a summary.

Taking demographics first, agents retire at the age of 65, which for many years was the Normal Retirement Age for Social Security in the U.S. Mortality rates are taken from the Human Mortality Database for the USA for 2016. We assume that all individuals die at the age of 100 if not before.

Turning to endowments, agents earn a wage in each period up to the age of 64 and receive Social Security payments from the age of 65. The shape of the deterministic age-productivity profile is taken from Harenberg and Ludwig (2019), who compute it from PSID data using the method in Huggett et al. (2011). This series is transformed so that average earnings are set at \$46,640, average US net earnings in 2016 (Social Security Administration, 2020). The values for the stochastic component of the income process are taken from Guvenen (2009), who reports an autoregressive parameter $\rho = 0.988$ and a variance of persistent shocks of $\sigma_v^2 = 0.015$. Public pensions, y^R , are set at 40 percent of the average earnings, which is approximately the average replacement rate afforded by Social Security (Biggs and Springstead, 2008). We assume that agents enter the model with no assets, $s_{-1} = 0$ and $b_{-1} = 0$.

Turning to the features of our asset market, we follow Campbell and Viceira (2002). The gross risk-free return is set at $R^f = 1.02$. We set the equity premium to $\omega = 4\%$ and stock volatility is $\sigma_\nu = 15.7\%$. These values represent common choices in the lifecycle literature (see, for example, Lusardi et al., 2017). We set the administrative load (δ) to 10% (as in Pashchenko, 2013, Lockwood, 2012, and

Table 1: Externally-set parameters

Parameter	Value	Source
<i>Demographics</i>		
Retirement date, T_R	45 (= 65 – 20)	SSA Historical Normal Retirement Age in US
Maximal life duration, T_M	80 (= 100 – 20)	Human Mortality Database, U.S. 2016
Cond. survival rates, $\{\pi_t\}$		
<i>Endowments</i>		
Average wage, \bar{y}	US\$ 46,640	Average net compensation 2016, SSA
Age productivity, $\{\mu_t\}$		Harenberg and Ludwig (2019)
Public pension, y^R	40% $\times \bar{y}$	Average SS replacement rate (Biggs and Springstead, 2008)
Labor income autocorr., ρ	0.988	Guvenen (2009)
Var. of persistent shocks, σ_v^2	0.015	Guvenen (2009)
<i>Asset Markets</i>		
Gross risk-free return, R^f	1.02	Campbell and Viceira (2002)
Equity premium, ω	4%	Campbell and Viceira (2002)
Stock volatility, σ_ν	15.7%	Campbell and Viceira (2002)
<i>Annuity Market</i>		
Min. annuity purchase, \underline{a}	US\$ 3,680	Pashchenko (2013)
Administrative load, δ	10%	Pashchenko (2013)

in the range of Brown, 2007). We set the minimum annuity purchase to \$3,680, calculated by converting the value in Pashchenko (2013) to 2016 dollars. The final feature of the asset market, F , the once-in-a-lifetime participation cost in the risky asset, is calibrated to match participation in stock markets – we defer discussion of that to below.

Preference parameters. We set the intertemporal elasticity of substitution to 0.5, so that its inverse is $\sigma = 2$, which is a common value in the literature. The time preference and risk aversion parameters, β and k , and the utility gap between life and death, u_l , are set so that lifecycle behaviors match our calibration targets, which we will detail later.

To set the bequest function parameters (θ , which governs the intensity of the bequest motive, and \bar{x} , which governs the extent to which bequests are a luxury good), we use the estimates of De Nardi et al. (2010), who study the problem of bequests in detail. Their model, which features additive preferences, implies values

for the IES and the discount factor that differ from ours, and we therefore cannot directly use their bequest parameters. Our approach is to replicate two targets implied by their estimates: (i) the maximal wealth at which no bequest is left; and (ii) the marginal propensity to consume wealth, both of which are computed for a living agent at the maximal age – such that she dies for sure in the next period – who can only save in the riskless asset. This removes any risk in the model and the risk-sensitive model reduces to the additive one. The parameters of De Nardi et al. (2010) imply a value of \$36,000 (in 1998 dollars) for the maximal no-bequest wealth and of 0.88 for the marginal propensity to bequeath wealth in the last period of life.¹⁹ To match these values, we set $\bar{x} = 8.50$ and $\theta = 56.55$.²⁰ See Appendix C.2 for further details on the calculation of these numbers.

Calibration of remaining parameters. Four parameters are calibrated to match features of behavior over the lifecycle. These parameters are the utility gap between life and death (u_l), the discount factor (β), the parameter governing risk aversion (k), and the cost of participating in stock markets (F). We calibrate them by matching four targets: the VSL at age 45, asset holdings at age 65, annuity holdings at 65 and the proportion of people at 65 who hold risky assets.²¹ For the

Table 2: Calibration targets

Target	Value	Source
VSL	\$10m	Kniesner and Viscusi (2019)
Mean wealth at age 65	\$366,000	Survey of Consumer Finances
Proportion holding annuities at age 65	5%	Pashchenko (2013)
Stock market participation rate at age 65	50%	Alan (2006)

Notes: See the text for calculation and further details on source.

VSL at age 45, we target a mean value of \$10m, which is suggested as a central estimate for the US by the recent review article by Kniesner and Viscusi (2019). This is close to the value of \$9.6m used by the US Department of Transportation (2016) and in the range of \$8.8m–\$10.6m, used by the Environmental Protection Agency (2011) in evaluating the Clean Air Act.

¹⁹These numbers are implied by the estimates in column 3 of Table 3 of their paper. See the discussion in Appendix D of their paper.

²⁰ \bar{x} is expressed in units of average income. The dollar equivalent is \$396,477.

²¹For the value of life we choose its estimated value at age 45 (and not 65) as a calibration target, as most empirical studies rely on wage-risk trade-offs and are estimated on samples comprising those of working age.

To obtain a target to match mean modeled wealth at the age of 65, we use the 2016 wave of the Survey of Consumer Finances (Federal Reserve Board, 2016). We select a sample of single individuals without children. There are only approximately 100 such individuals at each age – and so to increase the sample size we calculate the mean for those between 60 and 69. We winsorize at the 1st and 99th percentiles. This yields a mean of \$366,000. We also show in Appendix D.2 the robustness of our results to matching mean wealth at 45 instead of age of 65.

We target the proportion of households holding annuities. This proportion in the U.S. is extremely low (this fact is, of course, the source of the annuity ‘puzzle’). Lockwood (2012) reports that 3.6% of single retirees hold annuities; Pashchenko (2013) reports 5%, also for a sample of single retirees. We target the latter, slightly higher, number.

Finally, once-in-a-lifetime participation cost is calibrated (jointly with the preference parameters) to deliver a stock market participation rate of 50% at age 65, which is in the range reported by Alan (2006).

In our results, we will compare our simulated profiles to an additive model that will also be calibrated to meet these targets. As the risk aversion parameter k is constrained to equal zero in the additive specification, matching the targets requires us to add one degree of freedom. To do this, rather than imposing the 10% administrative load (δ), informed by the literature, we find the administrative load that would rationalize annuity demand.

4.5 Results

Parameter estimates. Table 3 gives our baseline parameter estimates and also shows, for comparison, the parameter estimates for the additive model. The estimate for our risk-sensitive model of β , 0.966, is close to values typically estimated in lifecycle models and additionally is very close to the value (0.97) assumed by Pashchenko (2013) and Lockwood (2012), two papers which try to rationalize low annuity-demand. De Nardi et al. (2010), on whose estimates we base our parameterization of the bequest function, also use a value of 0.97.

The specific values we obtain for u_l and k depend on normalization choices we made, and are best understood by thinking of the trade-off between consumption and life duration (for u_l) or by looking at how agents compare different lotteries over life duration (for k). Regarding u_l , consider a setting with an agent whose consumption equals the average annual income. Such an agent endowed with the calibrated value of u_l would be willing to give up about 7% of her consumption during her last year of life in exchange for one extra week of life.

The value of k is set to 0.867. The value of this parameter cannot, of course, be directly compared to coefficients of risk aversion in EZW models: k is only one of the determinants of risk aversion with respect to consumption (that is defined as $-c_t \frac{\partial^2 V_t}{\partial c_t^2} / \frac{\partial V_t}{\partial c_t}$) that is also driven by other preference parameters, such as σ (as in the additive model). For an interpretation of the magnitude of k , consider the following situation. A 65 year-old agent, endowed with the set of preferences estimated in our baseline model is faced with a previously unanticipated option to undergo some surgery, knowing that the surgery would increase her life expectancy, but would involve taking a 5% risk of an immediate death. With our estimated value of k , an agent would opt for surgery only if the increase in life expectancy (taking account of the risk of dying in the operation) exceeds 12 months.²² Had we set $k = 0$ (and kept all other parameters the same), the agent would opt for the operation when the increase in life expectancy is as short as 4 months.

The once-in-a-lifetime participation cost, at 170% of average annual income, is necessarily large to match observed (non-)participation in risky assets when the equity premium is 4%, (see, for example, Mehra and Prescott, 1985 and Kocherlakota, 1996). However, note that this is paid only once per life.²³ To place this quantity in perspective, for those who pay the participation cost, it represents 2.4% of lifetime consumption (discounted by the risk-free rate).²⁴

Estimated profiles. Figure 1 shows profiles of mean consumption, wealth, participation in stock market and VSL over the lifecycle for a simulated sample of individuals. Results are reported for three specifications. The first two correspond to the calibrated versions of the risk-sensitive (referred to as RS) and additive (referred to as “calibrated-additive”) models, whose parameters are shown in Table 3. Both models, by construction, match the quantitative targets which are marked

²²To calculate this, we find the scalar α that, when multiplied by all survival probabilities for ages after 65, exactly compensates agents, in terms of (ex-ante) utility, for the loss in utility associated with the additional 5% chance of dying at the age of 65. We can then use this quantity to calculate the new life expectancy taking into account both the risk of dying in the operation and the greater survival in each period if she survives.

²³In Section D, we show that our results are robust to participation costs being paid every period an agent invests in stocks.

²⁴This value is consistent with the literature. For instance, Favilukis (2013) sets a cost that is a combination of once-per-lifetime and once-per-period payments. The one-per-life cost is 2.1% of median wealth and the per-period cost 10% of this value. However, this cost has to be put in the context of the low discount factor ($\beta = 0.78$) and a low IES ($\sigma = 10$) of that paper’s calibration. In their baseline calibration with no catastrophic risk, Fagereng et al. (2017) report a per-period cost of approximately \$ 350 (1995 USD), but the participation rate exceeds 90%, despite a low discount factor and a low IES ($\beta = 0.827$ and $\sigma = 14.444$). Introducing catastrophic risk in that paper reduces the participation cost, albeit by diminishing the risk premium to a lower value than that we use in this paper. They further provide the outcomes of simulations with $\beta = 0.96$, $\sigma = 10$, and a per-period participation cost of \$ 300. In that case, the participation rate hits 100% at age 45 – highlighting the crucial role of β .

Table 3: Estimated parameters in baseline economy

Parameter	Risk-Sensitive	Additive Model
<i>Preferences</i>		
Inverse of IES, σ	2.0 [†]	2.0 [†]
Risk aversion parameter, k	0.867	0.000 [†]
Life-death utility gap, u_l	3.570	12.020
Discount factor, β	0.966	0.953
Bequest motive strength, θ	56.55 [†]	56.55 [†]
Bequest luxury good, \bar{x}	8.50 [†]	8.50 [†]
Annuity administrative load, δ	10% [†]	23%
<i>Asset Markets</i>		
Once-in-a-lifetime participation cost, F	170% of \bar{y}	175% of \bar{y}

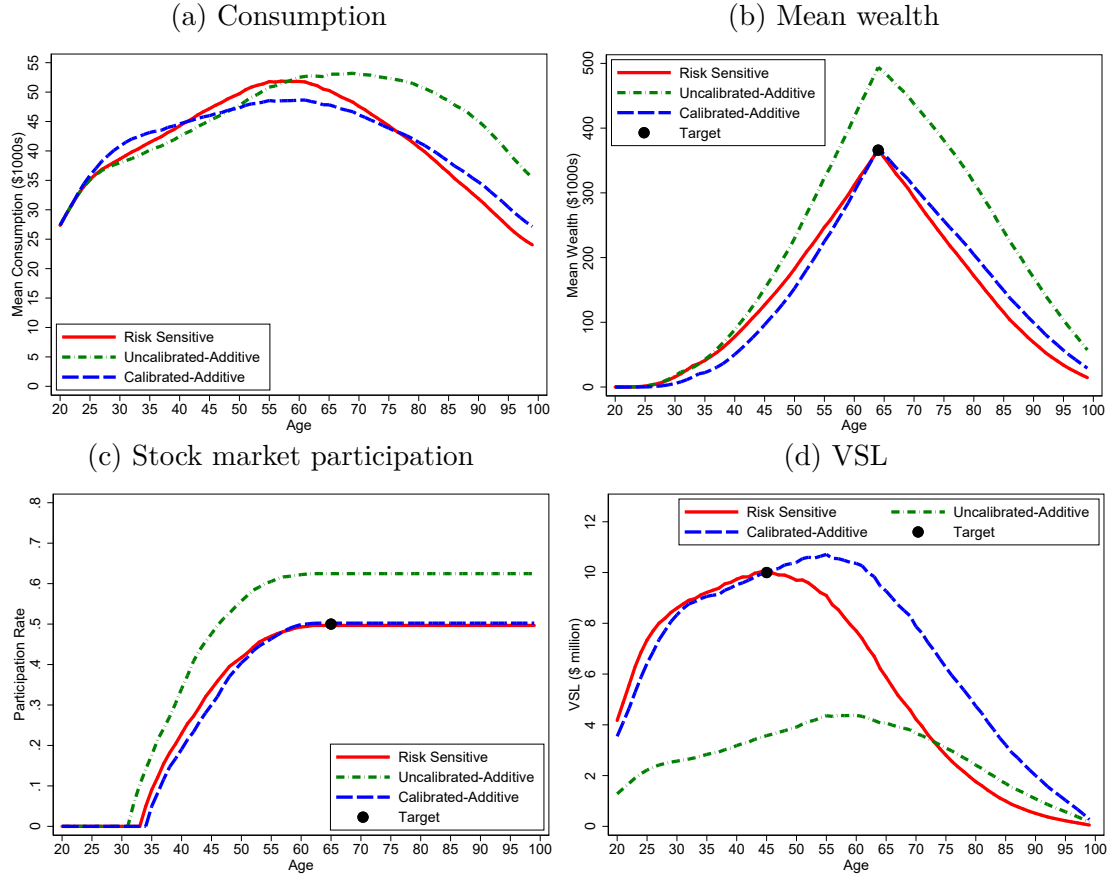
Notes: One unit of consumption is equal to \bar{y} . Quantities indicated by a [†] are imposed rather than estimated.

on the graphs and predict that the proportion of individuals holding annuities is 5%. The third specification, labeled as “uncalibrated-additive” corresponds to the model obtained while keeping all parameters of the risk-sensitive case fixed, but setting $k = 0$ to recover an additive specification. We show these results to facilitate comparative statics with respect to the risk aversion parameter k .

Comparative risk aversion. RS agents are more risk averse than the “uncalibrated-additive” agents, but are identical in all other aspects. Comparing the predictions of the RS model with those of the “uncalibrated-additive” model therefore reflects the theoretical predictions of Section 3 about the role of risk aversion. The framework here is, though, a richer one than that outlined in Section 3 – in particular with the addition of income and asset return uncertainty. These additions are likely to have important effects: income uncertainty generates a precautionary savings effect that is amplified with risk aversion.²⁵ The negative relationship between risk aversion and savings derived in Section 3 will thus be complemented by a precautionary effect that would imply an opposed relation. Uncertainty in asset returns may also contribute to a positive or negative relationship between risk aversion and savings, depending on the intertemporal elasticity of substitution and lifecycle income profile. The overall impact of risk aversion is therefore theoretically ambiguous, with its sign depending on the magnitude of the different risks at play. Figure 1 shows, however, that RS agents save less than

²⁵See Bommier and LeGrand (2019), who show, using risk-sensitive preferences in an infinite horizon setting, that there is a positive relationship between risk aversion and precautionary savings.

Figure 1: Lifecycle profiles for the baseline calibration



the “uncalibrated-additive” agents. Moreover RS agents are less likely to purchase annuities (the annuity participation rate is 5% for RS agents compared to 33% for additive agents). These are in line with the predictions of Proposition 1.

From a quantitative standpoint, this means that the effects of income and financial risks that we added in this quantitative investigation turn out to be too small to offset the effect of mortality risk highlighted in our theory section. The effect of the mortality risk tends therefore to dominate those of other risks. We can interpret this as an indication that mortality risks loom larger for individuals in their decision making than do the other risks they face.²⁶

Figure 1 shows also that RS agents have a higher VSL at all ages and are less likely to invest in stocks than the “uncalibrated-additive” agents. This simply reflects that risk aversion increases the willingness to reduce exposure to mortality and financial risks.

Overall, our results regarding the impact of risk aversion highlight that the more risk averse are individuals, the more they dislike taking risks of any kind,

²⁶This fact is also reflected in the very high willingness to pay for mortality risk reduction revealed by empirical studies, which we used to calibrate our model.

whether they are related to mortality, income or financial matters. It is worth noting, however, that our findings strongly contrast with those of well-known studies in the HF literature, such as Gomes and Michaelides (2005, 2008), who find a positive relationship between risk aversion and stock market participation, and Inkmann et al. (2011), who find that risk aversion increases the demand for annuities. Explanations for these differences are provided in Section 5.

Comparison of the calibrated models. Let us now compare the predictions of RS and “calibrated-additive” specifications. By construction, both specifications predict the same accumulated wealth, stock market participation and annuity market participation at age 65, and the same value of life at age 45. The lifecycle profiles for consumption, wealth and stock market participation are therefore similar (as are portfolio shares held in each asset), though with a divergence in the VSL over the second half of the lifecycle.²⁷

A fundamental difference between the specifications is in how they confront the ‘annuity puzzle’, that is, how they rationalize realistically low annuity demand. Low annuity demand is rationalized for additive agents with a counterfactually high administrative load (23% compared to 10% for the RS agents) and a relatively low discount factor (0.953 compared to 0.966 in the RS case). RS agents, on the other hand, are concerned that purchasing annuities may lead to a loss in case of an early death. Although they value the benefits of holding annuities to insure against the consumption needs in the case of a long life, they also want to retain significant investments in bonds or in stocks so that the early death adverse event is mitigated by the transmission of a bequest to her heirs. This allows a low level of annuity demand to be rationalized, even if annuities are priced at close to actuarially fair levels.

4.6 Does the precise VSL value matter?

In Appendix D, we provide an extended sensitivity analysis where we investigate the robustness of the results to the structure of the participation cost, the calibration age for the wealth and the target VSL. Here, we provide a brief discussion of the last of these – the impact of the VSL estimate. We know from our theoretical section that the choice of a plausible (and hence positive) value of the VSL is central to the relationship between risk aversion and household choices. A natural question, given that the empirical literature on VSL does not contain a consensus

²⁷Increasing risk aversion leads to a greater willingness to pay to avoid dramatic outcomes, such as death at young age, as compared to adverse but less dramatic outcomes, such as death at old age. Increasing risk aversion therefore tends to amplify the relationship between age and VSL.

estimate of its value, is how much our results would be impacted by choosing a VSL target in the upper or lower range of empirical estimates.

The value that we choose, \$10m, is approximately the level currently used by US policy-makers. To assess whether this particular choice is instrumental in the results, we estimate the parameters using the same calibration strategy and the same calibration targets as in Section 4.4, except that we consider two alternative targets for the VSL, \$7.5m and \$12.5m.

The calibrated parameters for the two VSL targets, as well as the plots of the related lifecycle profiles for consumption, wealth, VSL, and stock market participation can be found in Appendix D.1. These graphs show clearly that changing the VSL target has very little impact on individual decisions (consumption, wealth, asset market participation, and annuity purchase). The only significant difference between calibrations is on the VSL lifecycle profiles. This does not mean that VSL has no impact on the calibration in general – and in particular this does not contradict the theoretical findings of Section 3. The take-away is that the quantitative impact of the precise VSL target is limited when this target is very large, as the empirical evidence suggests. Choosing a much lower VSL target would noticeably impact our quantitative findings.

The reason that the precise VSL plays little role in agents' choices when the VSL is large is that the RS model admits a well-defined limit when the utility of life becomes infinite and mortality is assumed to be exogenous. To show this, we take the recursive equation (21) that defines the utility V_t , representing risk-sensitive preferences, and consider the limit where u_l tends to infinity while maintaining the product ku_l equal to a constant, which we denote by κ below. This involves taking the limit where VSL gets increasingly large while maintaining risk aversion with respect to the life duration constant. It can easily be shown through a first-order Taylor expansion that the risk-sensitive utility, V_t , can be approximated by $V_t \simeq \frac{1}{k}Y_t + X_t$, where X_t and Y_t are recursively defined by:

$$X_t = (1 - \beta)u(c_t) \tag{25}$$

$$+ \frac{\beta}{\pi_t + (1 - \pi_t)e^{Y_{t+1}}} \left(\pi_t E_t[X_{t+1}] + (1 - \pi_t)e^{Y_{t+1}} E_t[(1 - \beta)v(x_{t+1})] \right),$$

$$Y_t = (1 - \beta)\kappa - \beta \log \left(\pi_t E_t \left[e^{-Y_{t+1}} \right] + 1 - \pi_t \right). \tag{26}$$

Under exogenous mortality, Y_t is exogenous, and maximizing V_t is thus equivalent to maximizing X_t . For a large VSL, the RS model therefore converges to the model represented by X_t . Notice that the recursive equation that defines X_t has an additive structure, similar to the usual additive specification. However, it embeds an (exogenous) age-dependent discount factor $\frac{\beta}{\pi_t + (1 - \pi_t)e^{Y_{t+1}}}$ that reflects the impact

of mortality and risk aversion on impatience.²⁸

Since the results we obtain depend very little on the value of the VSL calibration target within the empirically relevant range, we deduce that our calibrated model is in fact relatively close to the infinite utility of life limit. In particular, as long as the VSL remains large, the uncertainty regarding the precise value of the VSL is of limited concern. It also means that, to a first-order approximation, the quantitative model we use could be replaced by a standard additive model with an age-dependent discount factor. This may be helpful for further applications.

5 Relation to previous studies using EZW preferences

Most papers in the HF literature rely on the additive specification, which lacks the flexibility to fully study the role of risk aversion in decision-making over the lifecycle. In such papers, the expression “risk aversion” is most often used to refer to a parameter (σ in our paper) that governs both intertemporal substitutability and risk preferences. It is, however, well understood that models that assume different values for the IES are not comparable in terms of risk aversion (see Kihlstrom and Mirman, 1974, for instance). The findings of those papers cannot therefore be compared to ours.

The relevant comparison is with the subset of papers, such as Gomes and Michaelides (2005, 2008), Inkmann et al. (2011) and many others, which, like us, use recursive preferences to study the role of risk aversion in isolation. A key difference is that our model was designed to fit empirical estimates of the VSL, without imposing preference homotheticity, while these papers typically use EZW preferences to obtain homothetic (and tractable) specifications without considering the implications for the VSL.

Formally, for EZW preferences, in the presence of bequests the recursion (7) defining the utility conditional on being alive becomes:

$$V_t = \left((1 - \beta)c_t^{1-\sigma} + \beta \left(E_t \left[\pi_t V_{t+1}^{1-\gamma} + (1 - \pi_t)\theta x_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}}, \quad (27)$$

where, as in (23), θ determines the intensity of the bequest motive.²⁹ There is typically no discussion of the value of mortality risk reduction in HF papers, as

²⁸While the model obtained when maximizing X_t features age-dependent time discounting, it is time consistent. Preferences are not stationary (they depend on age, reflecting the relationship between age and mortality), but they do not exhibit preference reversals.

²⁹A formal derivation of equation (27) can be found in the appendix of Gomes et al. (2009).

they assume that mortality is exogenous. Equation (27) nevertheless implicitly assumes a specific sign for the value of mortality risk reduction. In particular:

$$\frac{\partial V_t}{\partial \pi_t} = \beta \frac{E_t[V_{t+1}^{1-\gamma}] - \theta E_t[x_{t+1}^{1-\gamma}]}{1-\gamma} \left(E_t \left[\pi_t V_{t+1}^{1-\gamma} + (1-\pi_t) \theta x_{t+1}^{1-\gamma} \right] \right)^{\frac{\gamma-\sigma}{1-\gamma}} V_t^\sigma,$$

which can be positive or negative. If $\gamma > 1$, as is assumed in the papers referenced above, a positive value of mortality risk reduction is obtained only if $\theta > \frac{E_t[V_{t+1}^{1-\gamma}]}{E_t[x_{t+1}^{1-\gamma}]}$. The results of Gomes and Michaelides (2005, 2008) and those of Inkmann et al. (2011) indicate that this condition does not hold (at least not always) in their simulations.³⁰ In particular, a negative value of mortality risk reduction is systematically obtained when there is no bequest motive ($\theta = 0$), a case considered in several instances in those papers. With a negative value of mortality risk reduction, risk aversion is found to amplify savings. This difference in saving behavior, in turn, generates differences in the propensity to pay the stock market participation cost. This explains why Gomes and Michaelides (2005, 2008) find that more risk averse agents tend to participate more frequently in the stock market. Moreover, with a negative value of mortality risk reduction, the risk of losing annuitized wealth in case of an early death is not seen as a major concern, as short lives are seen as good outcomes. This impacts the willingness to purchase annuities, which is found to increase with risk aversion (Inkmann et al., 2011). Overall, with a negative VSL, risk aversion is found to increase savings, stock market participation and annuity purchases, providing conclusions which are opposite to ours. This is fully in line with the theoretical results developed in Section 3, where we discussed the case of a (counterfactual) negative value of mortality risk reduction.

In the VoL literature, the recursive models of Hugonnier et al. (2013) and Córdoba and Ripoll (2017) do not account for bequest motives and thus do not investigate the trade-offs between annuity and bond purchases. Savings are found to decrease with risk aversion (in line with our findings) when the IES is above 1, but the relationship is opposite when the IES is below 1. The reason for these opposite findings is not related to the sign of the VSL (which is positive in their analyses) but is a consequence of the non-monotonicity that we discussed in the savings example of Section 2.4. Those models imply a time-varying discount

³⁰One should notice, moreover, that if specification (27) were to be used with $\gamma > 1$ and a parameter θ large enough to generate positive values of mortality risk reduction, we would obtain a framework that is still non-monotone and where the intensity of the bequest motive would increase the willingness to pay for mortality risk reduction: $\frac{\partial^2 V_t}{\partial \pi_t \partial \theta} > 0$. However, this would go against intuition, since deriving utility from bequest reduces the welfare gap between life and death. In models such as the RS or additive models, which this paper argues are better suited to studying decisions in the face of mortality risk, altruism has a negative impact on the value of mortality risk reduction (see equations (38) and (39) for instance) – as one would expect.

factor proportional to $\pi_t^{\frac{1-\sigma}{1-\gamma}}$.³¹ This implies that when $\gamma < 1 < \sigma$, an agent who survives with probability $\pi_t < 1$ saves more than if she was sure to survive. This result echoes the monotonicity breakdown we emphasized in the savings example discussed in Section 2.4. As noted in discussing the example, the departure from monotonicity gets quite extreme when π_t gets small. Indeed if $\pi_t \rightarrow 0$, meaning that period t is almost surely the last period of life, then $\pi_t^{\frac{1-\sigma}{1-\gamma}}$ tends to ∞ . In this case, the propensity to consume in period t vanishes and the agent saves everything to consume in the subsequent period, even though she will almost certainly not live to see it. Notice that this tendency to over-save (as compared to non-dominated strategies) is also magnified when the risk aversion parameter γ increases towards one (while keeping $\sigma > 1$ and π_t fixed). Increasing risk aversion increases the extent to which dominated strategies are chosen. This drives the positive relationship between savings and risk aversion found in those papers.

Finally, an exception is the paper of Pashchenko and Porapakarm (2021), who study annuity demand with EZW preferences. The key difference between their approach and previous work is that they consider *non-homothetic* EZW preferences, which allow them to consider a positive value of life, an IES above one, and a risk aversion parameter greater than 1. Their findings are in line with ours and underline the role of considering a positive value of life for studying savings matters. However, non-homothetic EZW specifications, which are no more tractable than risk-sensitive preferences, retain the non-monotonicity of the general class of EZW preferences, which has the potential for complicating their use in investigations of the role of risk aversion.

6 Conclusion

Inspired by Samuelson (1937), economic contributions on intertemporal choice have most often relied on models that assume time-additive preferences. While time-additive preferences offered an elegant framework to formalize insightful theories, such as Modigliani’s lifecycle hypothesis, they have some serious limitations. One of the caveats associated with the time-additive model is its lack of flexibility, in particular, the fact that its use means that risk aversion and intertemporal substitutability cannot be disentangled. This was underlined both by theoreticians (Epstein and Zin, 1989) and experimentalists (Andersen et al., 2008). Theoretical contributions, such as those of Epstein and Zin (1989) and Weil (1990), have addressed this limitation by introducing a recursive framework that is both tractable

³¹See for example, from equation (15) in Córdoba and Ripoll (2017).

and flexible. These EZW preferences have met with a remarkable success. While EZW preferences were initially developed to deal with infinitely-lived agents, their adaptation to finite and random horizon settings encountered the serious difficulties that we discussed in Section 2.

In the current paper, we propose a framework that can model lifecycle behaviors with recursive preferences that are well-defined, flexible, monotone and can simultaneously match realistic (positive) values for mortality risk reduction and plausible lifecycle profiles for consumption and wealth. We outline how using such a specification facilitates new and intuitive insights on the role of risk aversion, affording a potential explanation for several documented – though imperfectly understood – household behaviors, such as the low demand for annuities.

In an era of on-going demographic changes, the economics of aging will remain a key research area, with new forms of risks becoming increasing sources of concern (e.g., the increased likelihood of expensive long-term care at the oldest ages). While time-additive preferences have facilitated valuable insights, we argue that the field would benefit from a framework that allows decisions in the face of survival risk to be studied in a setting where the role of risk aversion can be separated from that of intertemporal substitutability. The approach put forward in this paper could serve as a foundation of such a framework for both the HF and the VoL literatures.

Appendix

A Proofs

A.1 Proof of Proposition 1

Note that at the optimum, we must have $b_k > 0$ and $w_0 - b_k - a_k > 0$. We focus on the case where $a_k > 0$. The first-order conditions of the consumption problem (12) yields after some arrangement:

$$b_k + \frac{a_k}{\pi_0} = b_k e^{-\frac{k(1-\beta)}{\sigma}(u(R^f b_k + R^f \frac{a_k}{\pi_0}) - v(R^f b_k))}, \quad (28)$$

$$\frac{w_0 - b_k - a_k}{b_k + \frac{a_k}{\pi_0}} = (\beta(R^f)^{1-\sigma})^{-\frac{1}{\sigma}} \left(\pi_0 + (1 - \pi_0) e^{k(1-\beta)(u(R^f b_k + R^f \frac{a_k}{\pi_0}) - v(R^f b_k))} \right)^{\frac{1}{\sigma}}. \quad (29)$$

We define $a'_k = \frac{\partial a_k}{\partial k}$, $b'_k = \frac{\partial b_k}{\partial k}$, $\kappa = \frac{k(1-\beta)}{\sigma}(R^f)^{1-\sigma} > 0$, and $\Delta = k(1-\beta)(u(R^f b_k + R^f \frac{a_k}{\pi_0}) - v(R^f b_k))$ and obtain from (28) and (29):

$$\frac{b'_k + \frac{a'_k}{\pi_0}}{b_k + \frac{a_k}{\pi_0}} = \frac{b'_k}{b_k} - \kappa \left((b_k + \frac{a_k}{\pi_0})^{-\sigma} (b'_k + \frac{a'_k}{\pi_0}) - b_k^{-\sigma} b'_k \right) - \frac{\kappa}{k} \Delta, \quad (30)$$

$$\frac{b'_k + a'_k}{w_0 - b_k - a_k} + \frac{b'_k + \frac{a'_k}{\pi_0}}{b_k + \frac{a_k}{\pi_0}} = \frac{\left(\kappa (b_k^{-\sigma} b'_k - (b_k + \frac{a_k}{\pi_0})^{-\sigma} (b'_k + \frac{a'_k}{\pi_0})) + \frac{\kappa}{k} \Delta \right) (1 - \pi_0) e^\Delta}{\pi_0 + (1 - \pi_0) e^\Delta}. \quad (31)$$

Equations (30) and (31) yield:

$$b'_k = -\lambda_{b'_k} a'_k, \quad (32)$$

$$\text{with: } \lambda_{b'_k} = \frac{\frac{1}{w_0 - b_k - a_k} + \frac{1}{b_k + \frac{a_k}{\pi_0}} \frac{1}{\pi_0 + (1 - \pi_0) e^\Delta}}{\frac{1}{w_0 - b_k - a_k} + \frac{1}{b_k + \frac{a_k}{\pi_0}} \frac{\pi_0}{\pi_0 + (1 - \pi_0) e^\Delta} + \frac{1}{b_k} \frac{(1 - \pi_0) e^\Delta}{\pi_0 + (1 - \pi_0) e^\Delta}}, \quad (33)$$

which is positive for all values of Δ . We obtain from (32):

$$a'_k + b'_k = \frac{\frac{1}{b_k (b_k + \frac{a_k}{\pi_0})} \frac{a_k}{\pi_0} \frac{(1 - \pi_0) e^\Delta}{\pi_0 + (1 - \pi_0) e^\Delta}}{\frac{1}{w_0 - b_k - a_k} + \frac{1}{b_k + \frac{a_k}{\pi_0}} \frac{\pi_0}{\pi_0 + (1 - \pi_0) e^\Delta} + \frac{1}{b_k} \frac{(1 - \pi_0) e^\Delta}{\pi_0 + (1 - \pi_0) e^\Delta}} a'_k, \quad (34)$$

We also obtain from (32):

$$\frac{a'_k}{\pi_0} + b'_k = \lambda_{\frac{a'_k}{\pi_0} + b'_k} a'_k, \quad (35)$$

$$\lambda_{\frac{a'_k}{\pi_0} + b'_k} = \frac{\frac{1}{w_0 - b_k - a_k} \frac{1 - \pi_0}{\pi_0} + \frac{1}{b_k} \frac{(1 - \pi_0) e^\Delta}{\pi_0 + (1 - \pi_0) e^\Delta} \frac{1}{\pi_0}}{\frac{1}{w_0 - b_k - a_k} + \frac{1}{b_k + \frac{a_k}{\pi_0}} \frac{\pi_0}{\pi_0 + (1 - \pi_0) e^\Delta} + \frac{1}{b_k} \frac{(1 - \pi_0) e^\Delta}{\pi_0 + (1 - \pi_0) e^\Delta}} > 0.$$

Denoting by $U' = \frac{\partial}{\partial k} \left(\frac{c_{0,k}^{1-\sigma}}{1-\sigma} + \beta \frac{c_{1,k}^{1-\sigma}}{1-\sigma} \right)$, we obtain using (29), (34), and (35):

$$\frac{U'}{c_0^{-\sigma}} = \frac{\frac{1}{w_0 - b_k - a_k} \frac{1-\pi_0}{\pi_0} + \frac{1}{b_k} \frac{(1-\pi_0)^2 e^{2\Delta}}{\pi_0(\pi_0 + (1-\pi_0)e^\Delta)} + \frac{1}{b_k + \frac{a_k}{\pi_0}} \frac{(1-\pi_0)e^\Delta}{\pi_0 + (1-\pi_0)e^\Delta}}{\frac{1}{w_0 - b_k - a_k} + \frac{1}{b_k + \frac{a_k}{\pi_0}} \frac{\pi_0}{\pi_0 + (1-\pi_0)e^\Delta} + \frac{1}{b_k} \frac{(1-\pi_0)e^\Delta}{\pi_0 + (1-\pi_0)e^\Delta}} a'_k. \quad (36)$$

We therefore deduce from (32), (34), (35), and (36), that $-\frac{\partial b_k}{\partial k}$, $-\frac{\partial c_{0,k}}{\partial k}$, $\frac{\partial c_{1,k}}{\partial k}$, and $\frac{\partial}{\partial k} (u(c_{0,k}) + \beta u(c_{1,k}))$ have the same sign as $\frac{\partial a_k}{\partial k}$. Using (32) and (35) with (30), we obtain after substitution:

$$\left(\frac{\lambda \frac{b'_k + \frac{a'_k}{\pi_0}}{b_k + \frac{a_k}{\pi_0}} + \kappa \frac{\lambda \frac{b'_k + \frac{a'_k}{\pi_0}}{(b_k + \frac{a_k}{\pi_0})^\sigma} + \frac{\lambda b'_k}{b_k} + \kappa \frac{\lambda b'_k}{b_k^\sigma}}{a'_k} \right) a'_k = -\frac{\kappa}{k} \Delta, \quad (37)$$

which implies that $\frac{\partial a_k}{\partial k}$ has the same sign as $-\Delta$ (when $k > 0$). Since the sign of Δ is the same as the sign of $\frac{\partial V_0}{\partial \pi_0}$, this concludes the proof.

A.2 Deriving VSL expressions

We denote by $w_t = A_{t-1} + R^f b_{t-1} + R_t^s s_{t-1}$ the beginning-of-period wealth by $\omega_t = q_t a_t + b_t + s_t$ the total saving choice, by $\alpha_t^b = \frac{b_t}{\omega_t}$ the share in bonds and by $\alpha_t^s = \frac{s_t}{\omega_t}$ the share in stocks. The program of the alive agent can be rewritten as:

$$V_t(w_t, A_{t-1}, \eta_{t-1}, \zeta_{t-1}) = \max_{c_t \geq 0, \omega_t \geq 0, (\alpha_t^b, \alpha_t^s) \in [0,1]^2} (1 - \beta) u(c_t) - \frac{\beta}{k} \log \left(\pi_t E_t \left[e^{-kV_{t+1}(w_{t+1}, A_t, \eta_t, \zeta_t)} \right] + (1 - \pi_t) E_t \left[e^{-k(1-\beta)v(x_{t+1})} \right] \right),$$

subject to: $y_t + w_t = c_t + \omega_t + 1_{\eta_t=1} 1_{\eta_{t-1}=0} F$, $w_t = A_{t-1} + \omega_t \left(\frac{1}{q_t} + (R^f - \frac{1}{q_t}) \alpha_t^b + (R_{t+1}^s - \frac{1}{q_t}) \alpha_t^s \right)$, and $x_t = \omega_t (R^f \alpha_t^b + R_{t+1}^s \alpha_t^s)$.

The envelope theorem yields $\frac{\partial V_t}{\partial w_t} = \frac{\partial V_t}{\partial c_t} = (1 - \beta) u'(c_t)$. Using $\frac{\partial(1/q_t)}{\partial \pi_t} = -\frac{1}{\pi_t q_t}$ and $\frac{\omega_t}{q_t} (1 - \alpha_t^b - \alpha_t^s) = a_t$, after some manipulation, we get:

$$VSL_t = \frac{\beta}{1 - \beta} \frac{1}{c_t^{-\sigma}} \frac{\left(-\frac{E_t[e^{-kV_{t+1}(b_t, A_t, s_t, \eta_t, \zeta_t)}] - E_t[e^{-k(1-\beta)v(x_{t+1})}]}{k} \right)}{\pi_t E_t[e^{-kV_{t+1}(b_t, A_t, s_t, \eta_t, \zeta_t)}] + (1 - \pi_t) E_t[e^{-k(1-\beta)v(x_{t+1})}]} - \beta a_t \frac{E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} e^{-kV_{t+1}(b_t, A_t, s_t, \eta_t, \zeta_t)} \right]}{\pi_t E_t[e^{-kV_{t+1}(b_t, A_t, s_t, \eta_t, \zeta_t)}] + (1 - \pi_t) E_t[e^{-k(1-\beta)v(x_{t+1})}]}, \quad (38)$$

and in the additive case, by continuity for $k \rightarrow 0$:

$$VSL_t^{add} = \frac{\beta}{c_t^{-\sigma}} \left(\frac{V_{t+1}^{add}(b_t, A_t, s_t, \eta_t, \zeta_t)}{1 - \beta} - kv(x_{t+1}) \right) - \beta a_t E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \right]. \quad (39)$$

B Details on the computational implementation

There is no analytical solution to the agents' problem outlined in Section 4 of the main paper. We solve the model and obtain decision rules numerically and then use those decision rules to simulate the agents' behavior. The next two subsections describe the solution of the model and the simulation of decision rules.

B.1 Model solution

While alive, the agent maximizes her intertemporal utility by choosing a feasible allocation $(c_t, b_t, a_t, s_t, \eta_t)_{t \geq 0}$ in the set \mathcal{A} . The utility V_t of the alive agent at age t depends on five state variables: the beginning-of-period holdings in bonds b_{t-1} , annuities A_{t-1} and stocks s_{t-1} ; the stochastic component of labor income, ζ_{t-1} ; and the stock market participation status, $\eta_{t-1} \in \{0, 1\}$. The last of these is discrete, while the first four are continuous. Given that annuity purchase may only occur in period $T_R - 1$, we have $A_t = 0$ for all $t < T_R$ and $A_t = a_{T_R-1}$ for $t \geq T_R$. Since there exists a maximal age for the agent, T_M , we solve the model by iterating on the value function, starting from the last period of life. Utility maximization involves solving:

$$V_{T_M}(b_{T_M-1}, A_{T_M-1}, s_{T_M-1}, \eta_{T_M-1}, \zeta_{T_M-1}) = \max_{(c_{T_M}, b_{T_M}, a_{T_M}, s_{T_M}, w_{T_M}, \eta_{T_M}) \in \mathcal{A}} (1 - \beta)u(c_{T_M}) + E_t \left[e^{-k(1-\beta)v(x_{T_M+1})} \right], \quad (40)$$

subject to the constraints, which we don't restate here, outlined in Section 4 of the main paper. Due to the presence of stocks in bequest, the continuation utility in case of death is uncertain. Both the instantaneous utility function for period $T_m(u(x_{T_m}))$ and the utility obtained when dying and bequeathing $x_{T_{M+1}}(v(x_{T_{M+1}}))$ are known and the model is solved for a discrete set of points on a grid. This gives us knowledge of V_{T_m} at a subset of the points in the state space and allows us to approximate V_{T_m} as \hat{V}_{T_m} at all points. With this approximation in hand we solve an approximation to problem (40) for period $T_M - 1$ and then, iteratively, for all preceding periods t :

$$V_t(b_{t-1}, A_{t-1}, s_{t-1}, \eta_{t-1}, \zeta_{t-1}) = \max_{(c_t, b_t, a_t, s_t, w_t, \eta_t) \in \mathcal{A}} (1 - \beta)u(c_t) - \frac{\beta}{k} \log \left(\pi_t E_t \left[e^{-k\hat{V}_{t+1}(b_t, A_t, s_t, \eta_t, \zeta_t)} \right] + (1 - \pi_t) E_t \left[e^{-k(1-\beta)v(x_{t+1})} \right] \right). \quad (41)$$

Note that the only difference between the maximand in (41) and that in our household's problem is that the continuation value function is an approximate value function. We will now, briefly, discuss four features of the numerical procedure:

i) the discretization of the continuous variables, ii) the integration of the value function, iii) the approximation method for evaluating the value function at points outside the discretized state space and iv) how the optimization is carried out.

Discretization of the continuous variables. We define a variable, total liquid wealth, which is the sum of bond and stock holdings at the start of a period. We define a grid of 54 points from \$0 to \$10m such that the gaps between successive grid points are smaller at lower levels of wealth, where the curvature of the value function will be greater. We define a grid of 36 points for annuity income from \$0 to \$800,000 such that the gaps between successive grid points are smaller at lower levels of income. Earnings are placed on a grid of 7 points each year using the procedure introduced by Tauchen (1986).

Integration. There are three risks facing households: mortality, earnings and financial risks. The risks are independent. Realizations for the first of these are naturally discrete and integration involves a simple weighted average. For the latter two, we define a discrete set of possible realizations and integrate over outcomes using the Tauchen (1986) procedure.

Approximation. To evaluate the value function at points other than those in the discrete sub-set of points, we use linear interpolation in multiple dimensions.

Optimization. Every period households make up to four choices. They decide on how much to consume, how much to save in each of bonds and the risky asset, whether to pay the participation cost (if they have not previously done so) and how much of an annuity income stream to purchase (in the period before retirement). Our problem is not globally concave, so our optimization of the household's decision problem cannot fully rely on local approaches. We therefore start by discretizing the choice variables. We define three grids on the unit interval, S_a^t , S_s , S_c , which will represent shares of available resources dedicated to purchases of annuities, portfolio shares in stocks and consumption respectively. All grids have equally spaced nodes from 0 to 1 (except that in periods other than period T_{R-1} , the grid for annuities has only one element, 0, as no annuity purchase is possible in those periods). We evaluate the household's objective function at each combination of $(s_{a,i}^t)_{i=1,\dots,I^t}$ in S_a^t , $(s_{s,j})_{j=1,\dots,J}$ in S_s , and $(s_{c,k})_{k=1,\dots,K}$ in $S_{c,k}$. Defining the total available resources as $R_t = y_t + w_t$ (the sum of income and wealth) in time t , we set the annuity choice to $s_{a,i}^t R_t$, the level of saving in the risky asset set to $s_{s,j}(1 - s_{a,i}^t)R_t$, the level of consumption as $s_{c,k}(1 - s_{s,j})(1 - s_{a,i}^t)R_t$ and the

savings in the bond as $(1 - s_{c,k})(1 - s_{s,j})(1 - s_{a,i}^t)R_t$. We find the combination of the points that yields the maximum value to the household. This is our candidate optimal decision. We then do a further local search for the split between consumption and the bond using golden section search. Formally, taking the candidate maximum as indexed by s_{a,i^*}^t in S_a^t , s_{s,j^*} in S_s , s_{c,k^*} in $S_{c,k}$, we look for the utility-maximizing split between consumption and the bond in the interval for consumption of $[s_{c,\max\{1,k^*-1\}}(1 - s_{s,j^*})(1 - s_{a,i^*}^t)R, s_{c,\min\{K,k^*+1\}}(1 - s_{s,j^*})(1 - s_{a,i^*}^t)R]$.

For points where individuals have not already paid the participation charge we implement this procedure twice, once assuming they pay the charge, once assuming they do not. The maximum of these indicates the decision rule.

This procedure yields decision rules as a function of the vector of state variables \mathbf{X}_t : $\hat{c}(\mathbf{X}_t)_t, \hat{b}(\mathbf{X}_t)_t, \hat{s}(\mathbf{X}_t)_t, \hat{a}(\mathbf{X}_t)_t, \hat{\eta}(\mathbf{X}_t)_t$.

B.2 Simulation of profiles

Once decision rules $\hat{c}(\mathbf{X}_t)_t, \hat{b}(\mathbf{X}_t)_t, \hat{s}(\mathbf{X}_t)_t, \hat{a}(\mathbf{X}_t)_t, \hat{\eta}(\mathbf{X}_t)_t$ are obtained, we simulate a data set for 3,000 individuals. We do this as follows:

1. Initial values for wealth are set to 0.
2. Earnings draws for the first period of economic life are drawn randomly for each individual. Using these values of the state variables and the decision rules we can obtain optimal behavior in the first period.
3. We draw an equity price shock to apply to any equity holdings held at the end of the period.
4. Optimal behavior, the rate of return and the inter-temporal budget constraint yield the state variables for period 2.
5. We repeat steps (2) to (4) to obtain optimal behavior and subsequent state variables for each age up to 100. In most time periods, individuals will have realizations of the continuous state variables that are off this grid. Our approach here is to solve the individual's problem for decision rules as we do in the solution stage.

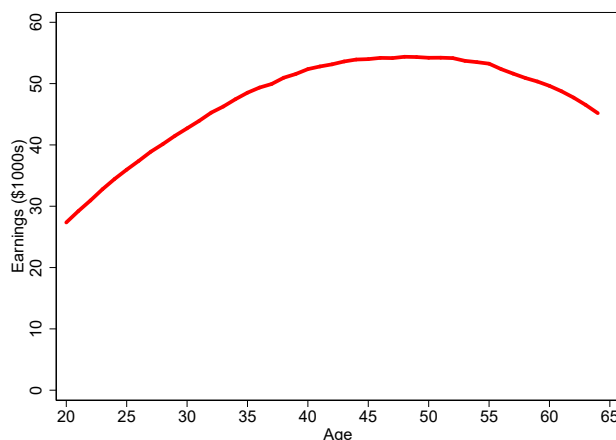
C Calibration details

C.1 Lifecycle productivity

The shape of the deterministic age-productivity profile is taken from Harenberg and Ludwig (2019), who compute it from PSID data using the method in Huggett

et al. (2011) US\$.³² Figure 2 displays the average earnings profile.

Figure 2: Average earnings over the lifecycle



C.2 Bequest parameters

To calibrate bequest parameters, we compute the marginal propensity to consume (MPC) and the maximum wealth at which no bequest will be left for an agent in the last period before certain death, when she can only invest in a riskless bond. Up to a factor $(1 - \beta)^{-1}$, her program is: $\max_{b \in \mathbb{R}} u(W - b) + \beta v(R^f b)$. There are two cases. (i) No bequest is left iff $u'(W) \geq \beta R^f v'(0)$. (ii) Otherwise, there is a positive bequest, determined by: $u'(W - b) = \beta R^f v'(R^f b)$, which implies that the maximal no-bequest wealth, denoted by W_0 , solves $u'(W_0) = \beta R^f v'(0)$, or using the expressions (22) and (23) for instantaneous utility functions:

$$W_0 = \kappa \bar{x}, \text{ with } \kappa = \left(\beta R^f \theta \right)^{-\frac{1}{\sigma}}. \quad (42)$$

If $W \geq W_0$, the agent's FOC implies for the optimal bequest b and for the MPC is:

$$b = \frac{W - \kappa \bar{x}}{1 + \kappa R^f} \text{ and } \frac{\partial c}{\partial W} = \frac{R^f \kappa}{1 + R^f \kappa}. \quad (43)$$

D Sensitivity analysis

D.1 High and low targets for the VSL

We calibrate the model using the same targets as in the baseline calibration, except that we consider in turn, a high VSL target of 12.5 million USD (+25% compared to the baseline) and a low VSL target of 7.5 million (−25% compared to the baseline).

³²Note that we multiply labor income with \bar{y} , thereby making sure that average labor income in 2016 is matched.

We report in Table 4 the value of parameters whose calibration is impacted compared

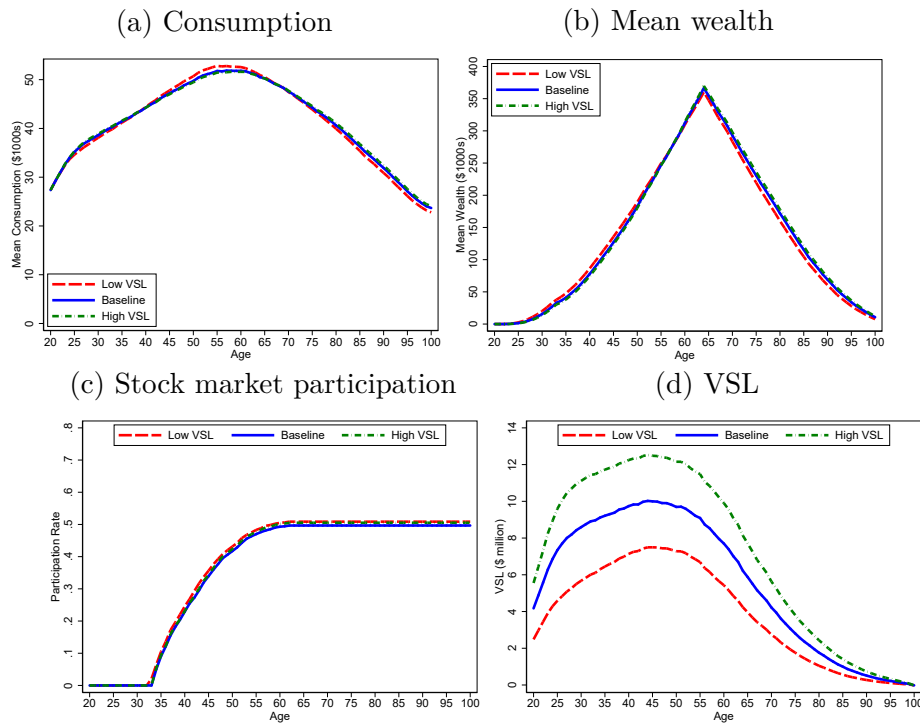
Table 4: Parameter calibration with low and high VSL targets

Parameter	Low VSL		High VSL	
	RS	Additive	RS	Additive
Risk aversion parameter, k	1.49	0.00 [†]	0.60	0.00 [†]
Life-death utility gap, u_l	2.30	9.03	4.81	15.01
Discount factor, β	0.965	0.953	0.966	0.953
Annuity admin. load, δ	10% [†]	23%	10% [†]	23%
Participation cost, F	161% of \bar{y}	175% of \bar{y}	167% of \bar{y}	175% of \bar{y}

Notes: One unit of consumption is equal to \bar{y} . Quantities indicated by a [†] are imposed rather than estimated.

to the baseline. Individual lifetime profiles are almost unchanged, as can be seen in Figure 3.

Figure 3: Lifecycle profiles for different VSL targets



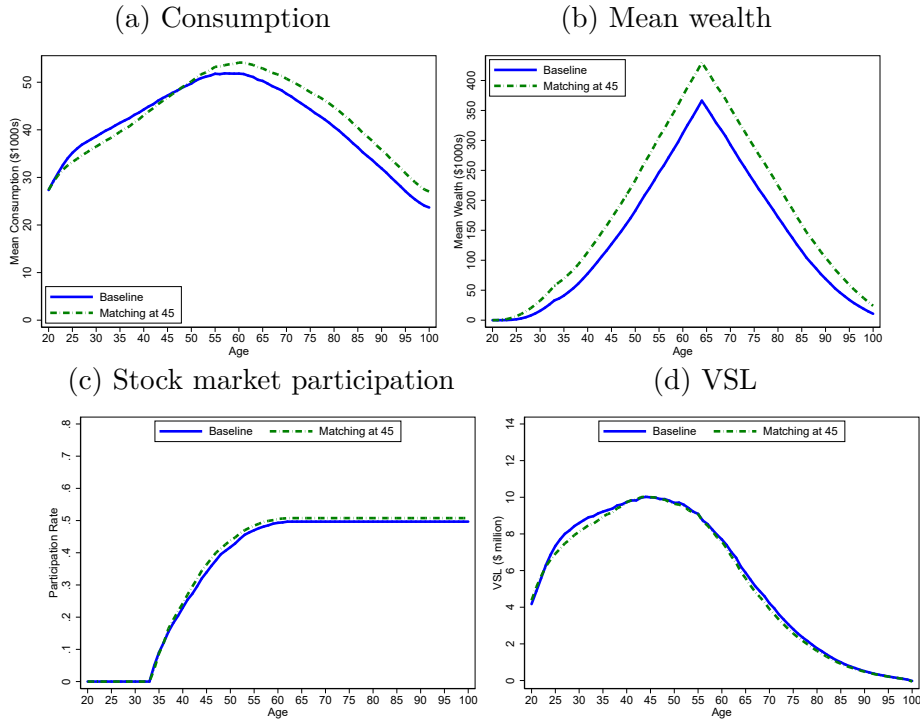
D.2 Changing the wealth target age from age 65 to age 45

In this subsection we show the sensitivity of our results to the calibration age for the wealth target, which we change from age 65 to age 45. All other targets remain unchanged (including the VSL). Table 5 shows the parameter values obtained in this calibration and allows them to be compared to the baseline.

Table 5: Parameter calibration with age 45 for the wealth target

Parameter	Risk-Sensitive	Additive Model
Risk aversion parameter, k	1.90	0.00 [†]
Life-death utility gap, u_l	2.49	9.68
Discount factor, β	0.977	0.976
Annuity administrative load, δ	10% [†]	54%
Participation cost, F	210% of \bar{y}	330% of \bar{y}

Figure 4: Lifecycle profiles for different targets regarding median wealth age



The main differences in this specification's calibration concern: (i) the annuity administrative load, δ (now required to be 54% in the additive model in order to match annuity demand, compared to the value of 23% of the baseline calibration); (ii) the participation cost, F (increases to 210%, from 170% for the RS model, but more substantially to 330% from 175% for the additive model). Targeting wealth at age 45 requires more substantial financial market imperfections than in the baseline calibration. This sensitivity exercise reinforces our conclusions in Section 4.5 regarding the comparison between RS and additive models.

Figure 4 reports the lifetime profiles for the baseline and for the case when age 45 is targeted. Quantitative differences are modest.

D.3 Participation cost every period

We here assume that the stock market participation cost is paid in every period in which an agent trades stocks. This follows Fagereng et al. (2017), among others. The budget constraint (14) when alive becomes:

$$c_t + b_t + s_t + F1_{s_t > 0} = y_t + R^f b_{t-1} + R_t^s s_{t-1}.$$

The rest of the agent’s program remains unchanged. The new values for the calibrated parameters are displayed in Table 6. The main difference compared to the baseline is the participation cost now falls from a one-off charge of 170% of annual income to annual cost of 14.2% of average income. Our substantive conclusions are unaffected by this choice.

Table 6: Parameter calibration with annual participation cost

Parameter	Risk-Sensitive	Additive Model
Risk aversion parameter, k	0.95	0.00 [†]
Life-death utility gap, u_l	3.50	12.273
Discount factor, β	0.967	0.953
Annuity administrative load, δ	10% [†]	23%
Annual Participation cost, F	14.2% of \bar{y}	14.4% of \bar{y}

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