Online Appendix to "Recursive Preferences, the Value of Life, and Household Finance"

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This online appendix is organized as follows:

1. Section S.1 provides a discussion of the implications of using EZW preferences in the context of uncertain lifetimes. We show that, unless the IES is assumed to be greater than 1 and risk aversion less than 1, EZW preferences generate implausible consumption paths, or implausible values for mortality risk reduction.

2. Section S.2 considers a series of computational experiments to better understand how the different risks of the model (mortality, income and asset return) interact with each other and contribute to our explanation of the relationships between risk aversion on the one hand and savings and annuity holdings on the other. This section focuses on risk-sensitive preferences.

3. Section S.3 contains two proofs: (i) the proof of Proposition 2, which is close to the proof of Proposition 1 provided in the main text, and (ii) the proof of translation invariance of risk-sensitive preferences. It also contains two other technical developments: (iii) the derivation of the VSL expressions in the risk-sensitive and additive setups, and (iv) the computation of the limits of the EZW model when IES or risk aversion parameter converges to 1.

4. Section S.4 presents the computational method used to solve the quantitative model.

To avoid confusion in the numbering of equations and sections between the main text and this supplemental online appendix, all numbers in this online appendix are prefixed by "S". Conversely, numbers without a prefix refer to an equation or a section in the main text.

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S.1 Using EZW preferences

In the absence of bequest motives, the standard homothetic EZW specification corresponds to the following recursion:

$$V_t = \left((1-\beta) c_t^{1-\sigma} + \beta \pi_t^{\frac{1-\sigma}{1-\gamma}} \left(E[V_{t+1}^{1-\gamma}] \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}}, \tag{S.1}$$

where $0 < \sigma \neq 1$ is the inverse of the IES and $0 < \gamma \neq 1$ is the risk aversion parameter (see Gomes and Michaelides, 2005 or Córdoba and Ripoll, 2017 among others). Note that the cases where $\sigma = 1$ can be deduced from (S.1) by continuity, while the case where $\gamma = 1$ and $\sigma \neq 1$ is either ill-defined, or with irrelevant properties. See Section S.3.4 below for mathematical derivations.

When $\gamma = \sigma$, the EZW model reduces to the additive model and when $\sigma = 1$, the EZW specification yields a special case of risk-sensitive preferences. In all cases where $\sigma \neq 1$ and $\sigma \neq \gamma$ the EZW model is not monotone with respect to first-order stochastic dominance.

As discussed in the main text, when $\frac{1-\sigma}{1-\gamma} < 0$, the recursion (S.1) may be ill-defined and admits $V_t = 0$ (independently of consumption choices) as the unique solution. A further discussion of the theoretical aspects can be found in Bommier et al. (2021).

Here, we ignore these convergence issues and focus on the model implications in terms of consumption and savings paths, as well as for the value of mortality risk reduction.[1] In this exercise, we make the following assumptions:

- the demographic aspects (mortality risk, retirement age, maximum age) are the same as in the main text;
- the labor income is risky and follows the same calibration as in the main text;
- agents can invest in riskless bonds and risky stocks, whose returns are the same as in the main text; there is no annuity;
- participation in the stock market is subject to a once-in-a-life cost, which is the same as in the main text;
- the time preference parameter is set to 0.953 and the IES to 1/2, as in the calibration for the additive model in our main analysis.

[1]Formally, if $\frac{1-\sigma}{1-\gamma} < 0$, we will assume that at the final date $T_{\max}$ (such that $\pi_{T_{\max}} = 0$) – which corresponds to 100 years in our data – we have $V_{T_{\max}} = (1-\beta)^{\frac{1}{1-\sigma}} c_{T_{\max}}$, while formally accounting for convergence issues would lead to $V_{T_{\max}} = 0$ (and thus $V_t = 0$ for all $t \leq T_{\max}$). This assumption is equivalent to assuming that the discount rate is set to zero in the final period.

Table 1: EZW calibration.

Parameter	Value	Source
Demographics		
Retirement date, T_R	47 (= 65 – 18)	SSA Historical Normal Retirement Age in US
Maximal life duration, T_M	82 (= 100 – 18)	
Cond. survival rates, $\{\pi_t\}$		US Life Tables for a male cohort born in 1940 (Bell and Miller, 2005)
Endowments		
Average wage, $\bar{y}$	US$ 43,104	Lifecycle average for 1940s men (own estimates)
Age productivity, $\{\mu_t\}$		Lifecycle average for 1940s men (own estimates)
Public pension, y^R	40%$\times\bar{y}$	Average SS replacement rate (Biggs and Springstead, 2008)
Labor income autocorr., ρ	0.977	Own estimates
Var. of persistent shocks, σ_υ^2	0.010	Own estimates
Asset Markets		
Gross risk-free return, R^f	1.02	Campbell and Viceira (2002)
Equity premium, ω	4%	Campbell and Viceira (2002)
Stock volatility, σ_ν	15.7%	Campbell and Viceira (2002)
Participation cost, F	123% of $\bar{y}$	Own calibration (additive model)
Preferences		
Inverse of IES, σ	2	Own calibration (additive model)
Discount factor, β	0.953	Own calibration (additive model)
Life-death utility gap, u_l	0	Homothetic preferences
Bequest motive strength, θ	0	No bequest motive
Bequest luxury good, $\overline{x}$	0	No bequest motive

Notes: One unit of consumption is equal to $\bar{y}$.

– there is no bequest motive: $\theta = \bar{x} = 0$.

We summarize the calibration in Table 1. As in the main text, the reported values assume that 1 unit of consumption corresponds to the average income $\bar{y}$.

EZW specifications represented as in (S.1) can be grouped in 4 (= 2 × 2) main categories depending on whether σ is less or greater than 1 and whether γ is less or greater than 1.[2] Whether $\sigma \lesseqgtr 1$ and $\gamma \lesseqgtr 1$ matter for two reasons. First, it

[2]We ignore here the cases with $\sigma = 1$, corresponding to additive or risk-sensitive preferences

determines the sign of the exponent $\frac{1-\sigma}{1-\gamma}$, and thus how the discount factor $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}}$ behaves at old ages when π_t becomes small. Second, it also determines the sign of $\frac{\partial V_t}{\partial \pi_t}$, and hence the sign of the value of mortality risk reduction. This can be summarized in Table 2.

Table 2: Discount factor $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}}$ and marginal utility of survival probability $\frac{\partial V_t}{\partial \pi_t}$ in the EZW model as a function of parameter values.

	$\sigma < 1$	$\sigma > 1$
$\gamma < 1$	$\beta\pi_t^{\frac{1-\sigma}{1-\gamma}} < 1$ $\frac{\partial V_t}{\partial \pi_t} > 0$	$\boldsymbol{\beta\pi_t^{\frac{1-\sigma}{1-\gamma}} \gg 1}$ **for small** $\boldsymbol{\pi_t}$ $\frac{\partial V_t}{\partial \pi_t} > 0$
$\gamma > 1$	$\boldsymbol{\beta\pi_t^{\frac{1-\sigma}{1-\gamma}} \gg 1}$ **for small** $\boldsymbol{\pi_t}$ $\boldsymbol{\frac{\partial V_t}{\partial \pi_t} < 0}$	$\beta\pi_t^{\frac{1-\sigma}{1-\gamma}} < 1$ $\boldsymbol{\frac{\partial V_t}{\partial \pi_t} < 0}$

In Table 2, we have bolded and shaded the properties that we consider to be major issues. This may related either to the discount factor $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}}$ which becomes very large when π_t is small (i.e. at old ages), or to the sign of the derivative $\frac{\partial V_t}{\partial \pi_t}$ which says whether agents associate a positive value to mortality risk reduction. As can be readily seen from Table 2, all cases but the one where both σ and γ are smaller than 1 have at least one problematic feature, generating either implausible patience patterns or negative value of mortality risk reduction (or both). To illustrate these issues, we plot the consumption and VSL profiles for four cases, corresponding to the four possibilities: $\sigma \lesseqgtr 1$ and $\gamma \lesseqgtr 1$. For IES, we consider the two values $\sigma = \frac{1}{2}$ and $\sigma = 2$, while for the risk aversion parameter, we consider $\gamma = \frac{2}{3}$ and $\gamma = 3$.[3]

The lifetime consumption paths are plotted in Figure 1. The consumption panels (b) and (c) suggest counterfactual consumption profiles, in which consumption remains low for most of the life-cycle, before rising sharply in old age. This is due to the fact that the parameters in panels (b) and (c) imply $\frac{1-\sigma}{1-\gamma} < 0$ and thus a discount factor $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}}$ that becomes large (and much larger than one) at old ages, when π_t is small. In these two specifications, mortality tends to reduce impatience, which makes them unsuitable for modeling consumption-saving behaviors. Cases (a) and (d) do not suffer from such drawbacks.

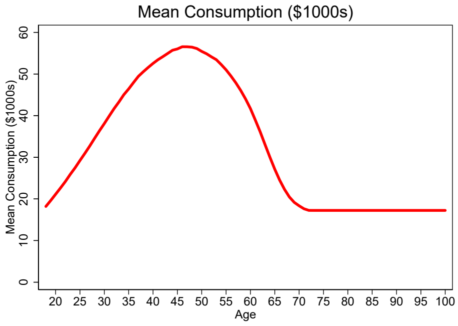
However, case (d) where both γ and σ are both greater than one predicts counterfactual negative values of mortality risk reduction. This can be seen in

that are already discussed in the main text. As explained in Section S.3.4, below, the cases with $\gamma = 1$ but $\sigma \neq 1$ are not well defined.

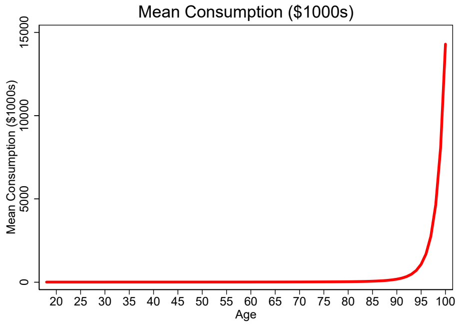
[3]We choose these values so that $\frac{1-\sigma}{1-\gamma}$ is never equal to 1, in which case the EZW model reduces to the additive model.

Figure 1: Consumption lifetime paths for the four parametrizations of the EZW model.

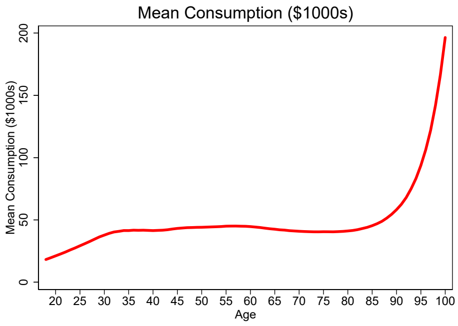
Case (a): $\sigma = \frac{1}{2}$ and $\gamma = \frac{2}{3}$



Case (b): $\sigma = 2$ and $\gamma = \frac{2}{3}$



Case (c): $\sigma = \frac{1}{2}$ and $\gamma = 3$



Case (d): $\sigma = 2$ and $\gamma = 3$

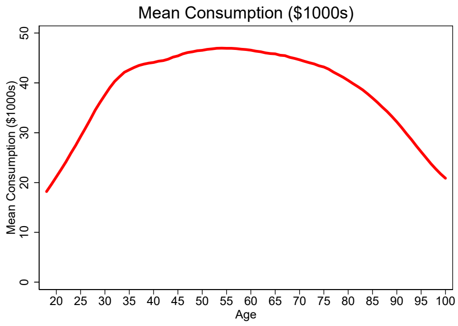
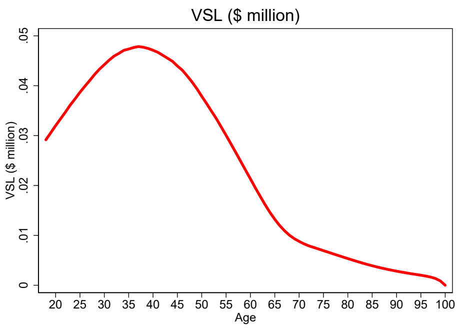


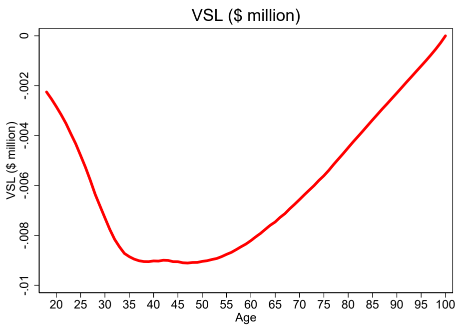
Figure 2, where we plot the VSL lifetime path for panels (a) and (d). We do not plot the paths for cases (b) and (c) which imply counterfactual consumption profiles. In case (d), VSL is negative at all ages, meaning that people would be willing to pay to shorten their lives.

Figure 2: VSL lifetime paths for two parametrizations of the EZW model.

Case (a): $\sigma = \frac{1}{2}$ and $\gamma = \frac{2}{3}$



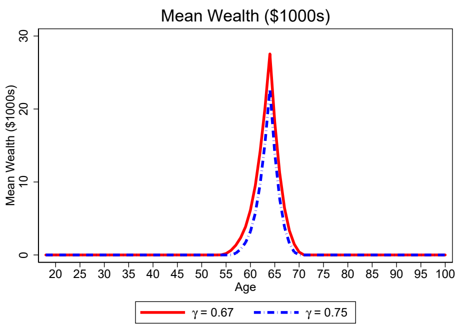
Case (d): $\sigma = 2$ and $\gamma = 3$



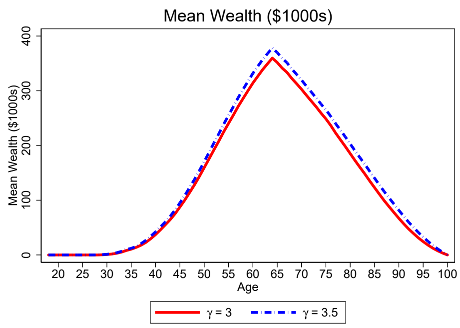
Even in the context of exogenous mortality, assuming a negative value of life has problematic consequences. Indeed, as we discussed in Section 3 of the main paper, the sign of the value of life determines the impact of risk aversion. To illustrate, in Figure 3, we focus on the cases where $\frac{1-\sigma}{1-\gamma} > 0$ (cases (a) and (d)) and plot the impact of an increase in risk aversion on savings profiles. In panel (a), where $\sigma = \frac{1}{2}$, we consider the effect of increasing the risk aversion parameter from $\gamma = \frac{2}{3}$ (value in Figures 1 and 2) to $\gamma = \frac{3}{4}$, for which we still have $\frac{1-\sigma}{1-\gamma} > 0$. In panel ($d$), where $\sigma = 2$, we consider an increase from $\gamma = 3$ from $\gamma = 4$ (value in Figures 1 and 2). Similarly to the theoretical predictions derived in Section 3 for risk-sensitive preferences, we see that when the value of life is positive (panel (a)), an increase in risk aversion leads to lower savings, while the opposite is true when the value of mortality risk reduction is negative (panel (d)). This, in turn, has implications for other aspects, such as stock market participation. Indeed, as we also report in Figure 3, we find that the stock market participation decreases with risk aversion when the value of life is positive (panel (a)), while stock market participation *increases* with risk aversion when the value of life is negative (in panel (d)). In other words, when the value of life is negative, more risk averse agents tend to participate *more* in the risky asset market.

Figure 3: Savings profiles and stock market participation paths for two cases, for the benchmark risk aversion parameter and a larger parameter value.

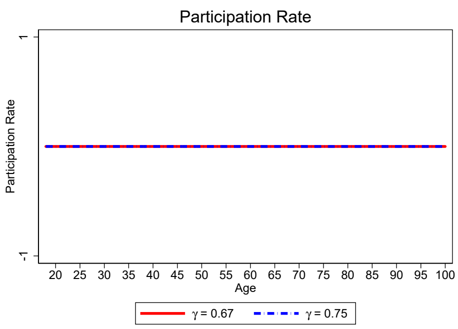
Case (a): $\sigma = \frac{1}{2}$ and $\gamma = \frac{2}{3}$ or $\gamma = \frac{3}{4}$ Case (d): $\sigma = 2$ and $\gamma = 3$ or $\gamma = 4$

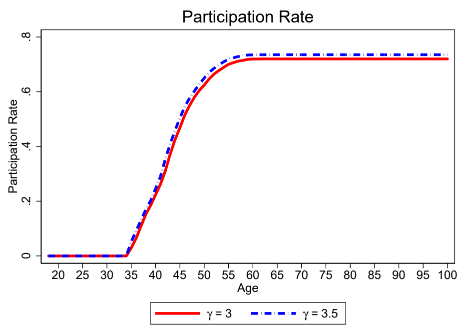


(a) Case (a): $\sigma = \frac{1}{2}$ and $\gamma = \frac{2}{3}$ or $\gamma = \frac{3}{4}$



(b) Case (d): $\sigma = 2$ and $\gamma = 3$ or $\gamma = 4$





Overall, $\sigma < 1$ and $\gamma < 1$ is the only case where the EZW specification does not generate counterfactual predictions. In such cases, consumption profiles are plausible, the value of mortality risk reduction is positive and the predictions about the role of risk aversion are similar to those obtained in the main body of the paper with risk-sensitive preferences. However, constraining σ to be less than one (and thus the IES to be greater than one) can be a non-trivial constraint. First, most of the literature provides estimates of the IES that are less than one (see Havránek, 2015). Moreover, a high IES is likely to lead to counterfactual large fluctuations in consumption over the life-cycle – which seems to be the case in the consumption profile of panel (a) of Figure 1.

S.2 Understanding the interplay between the different risks

We conduct a series of computational experiments to better understand how the risks and their interplay are shaping the role of risk aversion on savings and annuity holdings in the case of risk-sensitive preferences.

We consider three different risky environments. In the first one, agents face solely a mortality risk. There is no income risk or stock market participation. This simple model is a direct extension of our theoretical framework of Section 3 to the case of multiple periods and a finite value of mortality risk reduction. The second environment adds income risk to the previous setup. The last environment further allows stock market participation and therefore corresponds to our baseline model.

We investigate the role of risk aversion both in the small (variations between two positive values of k) and in the large (between $k = 0$ and $k > 0$). We therefore consider three values for the risk aversion parameter: $k = 0$ (additive model), baseline value of k ($k = 1.02$, as in our benchmark calibration), and a high value of k ($k = 1.25$, illustrating an increase in risk aversion).

The calibration is the same as in the baseline calibration, except that we possibly shut off one or both of two sources of risk: income risk and/or stock holdings. The value of the higher k is chosen such that the consumption profiles of the three models intersect (i.e., such that there is one age for which the consumption is the same in all models). The results can be summarized as follows.

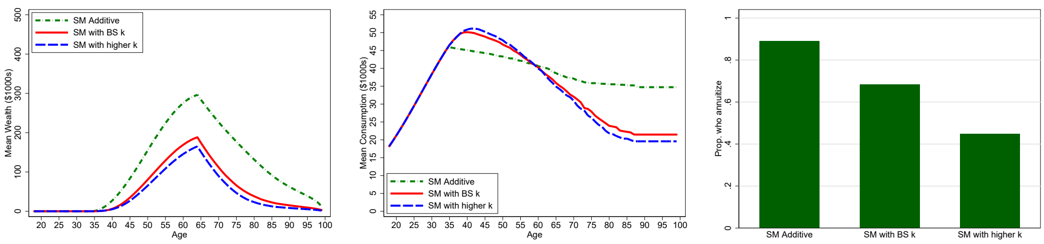
Case 1: There is no income uncertainty and no risky asset. We shut off the income risk and participation in the stock market. In the calibration, this implies setting income risk variances are zero and the stock market participation cost to infinity. Agents are heterogeneous in terms of income paths, just like when we assume income uncertainty, but they know ex-ante their (deterministic) income trajectories. They can only save in riskless bonds and annuities. The results are gathered in Figure 4, where we report the lifetime profiles related to savings and consumption (Figures 4a and 4b), as well as the proportion of agents holding annuities (4c). The acronym SM in graph labels stands for "simple model".

Risk aversion, be it in the small or in the large, has an unambiguous effect on savings and consumption. More risk averse agents tend to consume earlier in life and hence to save less. Conversely, they tend to consume less in late life. The consumption and savings in early age are the same for the three models because young agents are credit-constrained and simply live hand-to-mouth. The income being the same in all three models, the consumption profiles when credit-constrained are the same in the three models. Regarding annuities, the share of agents holding annuities decreases with risk aversion.

These results are in line with those of the theory of Section 3. What was formally shown to hold for an infinite value of mortality risk reduction in Proposition 2 tends to hold when assuming a finite value of mortality risk reduction.

Figure 4: Simple Model.

(a) Total Wealth. (b) Consumption. (c) Annuity.



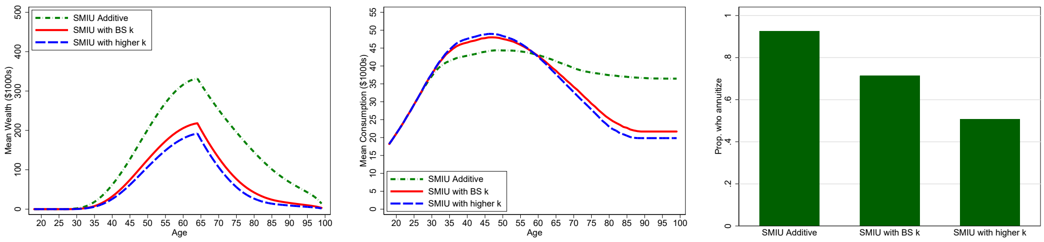
Case 2: There is no risky asset but there is realistically-calibrated income risk. We add income risk to the previous model. Stock market participation cost is still infinite, but income risk variances are set to their baseline values. The results are gathered in Figure 5, where we report the lifetime profiles related to savings and consumption (Figures 5a and 5b), as well as the proportion of agents holding annuities (5c).

The overall picture remains similar to the previous case. However, because of income uncertainty, agents in the three cases tend to save more than in the absence of income risks, reflecting a standard prudence effect. Hence, agents stop being constrained at slightly younger ages. The precautionary motives also tends to increase the proportion of agents holdings annuities, and the effect is stronger for more risk averse agents. In other words, the cross-effect of income risk and risk aversion on savings is positive.[4]

This case confirms that the well-known precautionary motive is at play in our model. However, its quantitative effect is rather modest and it does not offset the effect of mortality risk on savings. When income and mortality risks are combined, more risk averse agents still end up saving less. As in Case 1, annuity purchases fall with risk aversion.

Figure 5: Simple Model with Income Uncertainty.

(a) Total Wealth. (b) Consumption. (c) Annuity.



Case 3: There is a risky asset and income risk. This case corresponds to the baseline calibration (hence, the label BS). Results are plotted in Figure 6. The introduction of risky assets has an unambiguous effect on savings: all agents tend to have a higher mean wealth. Since risky assets offer on average a much higher return than riskless asset, participation to stock markets allow agents through the compounding effects to enjoy higher savings. The benefit of risky asset is higher when agents participate more often to stock markets and hold a higher share of their wealth in stocks. Both effects are more pronounced for less risk averse agents. This explains why additive agents see the largest change in wealth with the introduction of risky assets, while agents with higher k see the smallest change in wealth from it. The differences between lifetime savings profiles in Figure 6a are thus much more pronounced than in Figure 5a in the absence of risky assets.

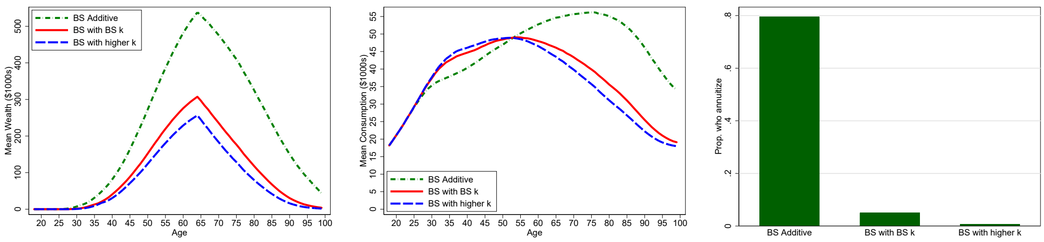
In addition to magnifying wealth heterogeneity generated by risk aversion heterogeneity, the introduction of risky assets also strongly impacts the demand for annuities. For all agents, annuities are crowded-out by stocks. The introduction of stocks reduces the demand for annuities. As can be seen in Figure 6c, the effect is stronger for more risk averse agents. The reduction in the proportion of agents annuitizing is modest for additive agents, as the share decreases by approximately 10 points compared to the SMIU. The reduction is much larger for risk averse agents (with baseline k) for whom the share of annuity holdings drops from around

[4]This cross-effect is not clear cut on total savings of Figure 4a because of the compounded wealth effects.

70% to 5%. The crowding out effect of stocks on annuities is thus very sizable for risk averse agents. Finally, for the most risk averse agents, they hold very few annuities.

Figure 6: Baseline Model.

(a) Total Wealth. (b) Consumption. (c) Annuity.



S.3 Additional proofs

This section contains two proofs and two technical developments. First, in Section S.3.1, we prove the result of Proposition 2. Second, in Section S.3.2, we prove the translation invariance property of risk-sensitive preferences. Third, in Section S.3.3, we provide the detailed derivation of the VSL expressions in the risk-sensitive and additive setups. Finally, in Section S.3.4, we discuss the limit cases where the IES or the risk aversion tends to 1 in EZW specifications.

S.3.1 Proof of Proposition 2

We consider the program of an agent choosing riskless savings and annuities to maximize her utility function given by the limit of the risk-sensitive utility function for $u_l \to \infty$, while holding the quantity $\kappa = ku_l$ constant. The program can be written as follows:

$$\begin{aligned} V_t^\infty(a_{t-1}, b_{t-1}) = \max_{a_t, b_t} \ & (1-\beta)u(y_t + R^f(b_{t-1} + \frac{a_{t-1}}{\pi_{t-1}}) - a_t - b_t) \\ & + \frac{\beta}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} \Big(\pi_t V_{t+1}^\infty(a_t, b_t) + (1-\pi_t)z_{\kappa,t+1}E_t[(1-\beta)v(R^f b_t)]\Big), \end{aligned} \tag{S.2}$$

where $(y_t)_t$ is the deterministic income path and we highlight the dependence of $z_{\kappa,t}$ in κ:

$$\log(z_{\kappa,t}) = (1-\beta)\kappa - \beta \log\left(\pi_t / z_{\kappa,t+1} + 1 - \pi_t\right). \tag{S.3}$$

We assume that the program admits an interior solution at all dates, denoted by $(a_{\kappa,t}, b_{\kappa,t})_t$. These solutions are given by the following first-order conditions:

$$
\begin{aligned}
& u'(y_t + R^f(b_{\kappa,t-1} + \frac{a_{\kappa,t-1}}{\pi_{t-1}}) - a_{\kappa,t} - b_{\kappa,t}) \\
&= \frac{\beta R^f}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} \Bigg(\pi_t u'(y_{t+1} + R^f(b_{\kappa,t} + \frac{a_{\kappa,t}}{\pi_t}) - a_{\kappa,t+1} - b_{\kappa,t+1}) \\
&\qquad + (1-\pi_t) z_{\kappa,t+1} v'(R^f b_{\kappa,t}) \Bigg), \\
&= \frac{\beta R^f}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} u'(y_{t+1} + R^f(b_{\kappa,t} + \frac{a_{\kappa,t}}{\pi_t}) - a_{\kappa,t+1} - b_{\kappa,t+1}),
\end{aligned}
$$

or

$$
\begin{aligned}
u'(y_t + R^f(b_{\kappa,t-1} + \frac{a_{\kappa,t-1}}{\pi_{t-1}}) - a_{\kappa,t} - b_{\kappa,t}) &= \frac{\beta R^f}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} \\
&\quad \times u'(y_{t+1} + R^f(b_{\kappa,t} + \frac{a_{\kappa,t}}{\pi_t}) - a_{\kappa,t+1} - b_{\kappa,t+1}), \\
u'(y_{t+1} + R^f(b_{\kappa,t} + \frac{a_{\kappa,t}}{\pi_t}) - a_{\kappa,t+1} - b_{\kappa,t+1}) &= z_{\kappa,t+1} v'(R^f b_{\kappa,t}), \\
u'(y_t + R^f(b_{\kappa,t-1} + \frac{a_{\kappa,t-1}}{\pi_{t-1}}) - a_{\kappa,t} - b_{\kappa,t}) &= \frac{\beta R^f z_{\kappa,t+1}}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} v'(R^f b_{\kappa,t}).
\end{aligned}
$$

We compute the derivatives of the FOCs with respect to κ. We denote the partial derivatives by a prime: $a'_{\kappa,t} = \frac{\partial a_{\kappa,t}}{\partial \kappa}$, $b'_{\kappa,t} = \frac{\partial b_{\kappa,t}}{\partial \kappa}$, and $z'_{\kappa,t} = \frac{\partial z_{\kappa,t}}{\partial \kappa}$. We also denote by $c_{\kappa,t} = y_t + R^f(b_{\kappa,t-1} + \frac{a_{\kappa,t-1}}{\pi_{t-1}}) - a_{\kappa,t} - b_{\kappa,t}$ the date-t consumption level. We obtain:

$$
\begin{aligned}
-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})}(b'_{\kappa,t} + a'_{\kappa,t}) &= -\frac{(1-\pi_t)z'_{\kappa,t+1}}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} \qquad \text{(S.4)} \\
&\quad + \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})}(R^f(b'_{\kappa,t} + \frac{a'_{\kappa,t}}{\pi_t}) - (b'_{\kappa,t+1} + a'_{\kappa,t+1})), \\
-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})}(b'_{\kappa,t} + a'_{\kappa,t}) &= \frac{\pi_t z'_{\kappa,t+1}}{z_{\kappa,t+1}(\pi_t + (1-\pi_t)z_{\kappa,t+1})} + R^f \frac{v''(R^f b_{\kappa,t})}{v'(R^f b_{\kappa,t})} b'_{\kappa,t}. \qquad \text{(S.5)}
\end{aligned}
$$

We prove by backward induction that $a'_{\kappa,t} < 0$, $b'_{\kappa,t} > 0$, and $b'_{\kappa,t} + a'_{\kappa,t} < 0$. At date $t = T_{\max} - 1$, we have $b'_{\kappa,t+1} = a'_{\kappa,t+1} = 0$ and the result follows from the two-period model. In particular, we have $b'_{\kappa,T_{\max}-1} + a'_{\kappa,T_{\max}-1} < 0$. We assume that the result holds at some date $t+1$, such that $b'_{\kappa,t+1} + a'_{\kappa,t+1} < 0$.

Combining (S.4) and (S.5) to remove $z'_{\kappa,t+1}$, we obtain:

$$b'_{\kappa,t} = -\lambda_{b'_{\kappa,t}} a'_{\kappa,t} + \mu_{b'_{\kappa,t}} (b'_{\kappa,t+1} + a'_{\kappa,t+1}), \tag{S.6}$$

$$\text{where: } \lambda_{b'_{\kappa,t}} = \frac{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})}}{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{\pi_t R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t)z_{\kappa,t+1}R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}} > 0, \tag{S.7}$$

$$\mu_{b'_{\kappa,t}} = \frac{-\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} \frac{\pi_t}{\pi_t+(1-\pi_t)z_{\kappa,t+1}}}{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{\pi_t R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t)z_{\kappa,t+1}R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}} > 0. \tag{S.8}$$

We then use (S.6) and (S.7) to obtain:

$$\frac{a'_{\kappa,t}}{\pi_t} + b'_{\kappa,t} = \lambda_{\frac{a'_{\kappa,t}}{\pi_t}+b'_{\kappa,t}} a'_{\kappa,t} + \mu_{b'_{\kappa,t}} (b'_{\kappa,t+1} + a'_{\kappa,t+1}), \tag{S.9}$$

where:

$$\lambda_{\frac{a'_{\kappa,t}}{\pi_t}+b'_{\kappa,t}} = \frac{\frac{1-\pi_t}{\pi_t}\left(-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{z_{\kappa,t+1}R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}\right)}{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{\pi_t R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t)z_{\kappa,t+1}R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}} > 0. \tag{S.10}$$

Combining (S.9) and (S.10) yields:

$$R^f(b'_{\kappa,t} + \frac{a'_{\kappa,t}}{\pi_t}) - (b'_{\kappa,t+1} + a'_{\kappa,t+1}) = R^f \lambda_{\frac{a'_{\kappa,t}}{\pi_t}+b'_{\kappa,t}} a'_{\kappa,t} - \mu_{c'_{\kappa,t}} (b'_{\kappa,t+1} + a'_{\kappa,t+1}), \tag{S.11}$$

where:

$$\mu_{c'_{\kappa,t}} = \frac{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{(1-\pi_t)z_{\kappa,t+1}R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}}{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{\pi_t R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t)z_{\kappa,t+1}R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}} > 0. \tag{S.12}$$

The difference of (S.4) and (S.5) implies:

$$0 = \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})}(R^f(b'_{\kappa,t} + \frac{a'_{\kappa,t}}{\pi_t}) - (b'_{\kappa,t+1} + a'_{\kappa,t+1})) - \frac{z'_{\kappa,t+1}}{z_{\kappa,t+1}} - R^f \frac{v''(x_{t+1})}{v'(x_{t+1})} b'_{\kappa,t},$$

which using (S.11) leads to:

$$a'_{\kappa,t} = \frac{-\frac{z'_{\kappa,t+1}}{z_{\kappa,t+1}}}{-\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} R^f \lambda_{\frac{a'_{\kappa,t}}{\pi_t}+b'_{\kappa,t}} - R^f \frac{v''(x_{t+1})}{v'(x_{t+1})} \lambda_{b'_{\kappa,t}}} + \frac{-\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} \mu_{c'_{\kappa,t}} - R^f \frac{v''(x_{t+1})}{v'(x_{t+1})} \mu_{b'_{\kappa,t}}}{-\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} R^f \lambda_{\frac{a'_{\kappa,t}}{\pi_t}+b'_{\kappa,t}} - R^f \frac{v''(x_{t+1})}{v'(x_{t+1})} \lambda_{b'_{\kappa,t}}} (b'_{\kappa,t+1} + a'_{\kappa,t+1}). \tag{S.13}$$

Equation (S.3) implies that $\frac{z'_{\kappa,t+1}}{z_{\kappa,t+1}} > 0$. Thus, using the induction hypothesis, we have from (S.13) that $a'_{\kappa,t} < 0$ and from (S.11) that $\frac{a'_{\kappa,t}}{\pi_t} + b'_{\kappa,t} < 0$.

We also have from (S.6):

$$a'_{\kappa,t} + b'_{\kappa,t} = \frac{-\frac{(\pi_t-1)R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}}\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t)z_{\kappa,t+1}R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}}\frac{v''(x_{t+1})}{v'(x_{t+1})}}{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{\pi_t R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}}\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t)z_{\kappa,t+1}R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}}\frac{v''(x_{t+1})}{v'(x_{t+1})}} a'_{\kappa,t}$$
$$- \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})}\pi_t(b'_{\kappa,t+1} + a'_{\kappa,t+1}),$$

which with $a'_{\kappa,t} < 0$ implies $a'_{\kappa,t} + b'_{\kappa,t} < 0$.

We also have from (S.5) and (S.13):

$$-R^f \frac{v''(x_{t+1})}{v'(x_{t+1})} b'_{\kappa,t} = \frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})}(b'_{\kappa,t} + a'_{\kappa,t}) + \frac{\pi_t z'_{\kappa,t+1}}{z_{\kappa,t+1}(\pi_t + (1-\pi_t)z_{\kappa,t+1})} > 0,$$

implying $b'_{\kappa,t} > 0$.

S.3.2 Proof of translation invariance in the risk-sensitive model

We recall that the recursion characterizing the utility function representing risk-sensitive preferences in the presence of bequest can be written as follows:

$$V_t = (1-\beta)u(c_t) - \frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-kV_{t+1}}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]\right), \quad \text{(S.14)}$$

where we use the same notation as in the main text.

We will prove the following lemma.

Lemma 1 *Consider the specification (S.14) and assume that $v(0)$ is finite. Simultaneously changing the instantaneous utility function for consumption to $u(c) - (1-\beta)v(0)$ and that for bequest to $v(x) - v(0)$ produces a simple shift in utilities (V_t is changed to $V_t - (1-\beta)v(0)$) that has no effect on preferences.*

In other words, Lemma 1 implies that if the utility of dying and leaving no bequest is finite, we can normalize v by setting $v(0) = 0$ without loss of generality. Note that the results also hold when there is no bequest motive ($v(x)$ equals a finite constant u_d independent from x).

Proof. Observe that we have:

$$\log\left(\pi_t E_t\left[e^{-kV_{t+1}}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]\right)$$
$$= \log\left(e^{-k(1-\beta)v(0)}\left(\pi_t E_t\left[e^{-k(V_{t+1}-(1-\beta)v(0))}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)(v(x_{t+1})-v(0))}\right]\right)\right),$$
$$= -k(1-\beta)v(0) + \log\left(\pi_t E_t\left[e^{-k(V_{t+1}-(1-\beta)v(0))}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)(v(x_{t+1})-v(0))}\right]\right).$$

Hence, we deduce:

$$V_t - (1-\beta)v(0) = (1-\beta)u(c_t) + \beta(1-\beta)v(0) - (1-\beta)v(0)$$
$$- \frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-k(V_{t+1}-(1-\beta)v(0))}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)(v(x_{t+1})-v(0))}\right]\right),$$
$$= (1-\beta)(u(c_t) - (1-\beta)v(0))$$
$$- \frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-k(V_{t+1}-(1-\beta)v(0))}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)(v(x_{t+1})-v(0))}\right]\right).$$

Defining $\tilde{V}_t = V_t - (1-\beta)v(0)$ and $\tilde{u}(c) = u(c) - (1-\beta)v(0)$, we obtain that the preferences represented by V_t may also be represented by $\tilde{V}_t$ defined by the following recursion:

$$\tilde{V}_t = (1-\beta)\tilde{u}(c_t) - \frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-k\tilde{V}_{t+1}}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)(v(x_{t+1})-v(0))}\right]\right),$$

which completes the proof. ■

S.3.3 Deriving VSL expressions

We denote by $w_t = A_{t-1} + R^f b_{t-1} + R^s_t s_{t-1}$ the beginning-of-period wealth by $\omega_t = q_t a_t + b_t + s_t$ the total saving choice, by $\alpha^b_t = \frac{b_t}{\omega_t}$ the share in bonds and by $\alpha^s_t = \frac{s_t}{\omega_t}$ the share in stocks. The program of the alive agent can be rewritten as:

$$V_t(w_t, A_{t-1}, \eta_{t-1}, \zeta_{t-1}) = \max_{c_t\geq 0, \omega_t \geq 0, (\alpha^b_t, \alpha^s_t)\in[0,1]^2} (1-\beta)u(c_t)$$
$$-\frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-kV_{t+1}(w_{t+1}, A_t, \eta_t, \zeta_t)}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]\right),$$

subject to: $y_t + w_t = c_t + \omega_t + 1_{\eta_t=1}1_{\eta_{t-1}=0}F$, $w_t = A_{t-1} + \omega_t(\frac{1}{q_t} + (R^f - \frac{1}{q_t})\alpha^b_t + (R^s_{t+1} - \frac{1}{q_t})\alpha^s_t)$, and $x_t = \omega_t(R^f\alpha^b_t + R^s_{t+1}\alpha^s_t)$.

The envelope theorem yields $\frac{\partial V_t}{\partial w_t} = \frac{\partial V_t}{\partial c_t} = (1-\beta)u'(c_t)$. Using $\frac{\partial(1/q_t)}{\partial \pi_t} = -\frac{1}{\pi_t q_t}$ and $\frac{\omega_t}{q_t}(1-\alpha^b_t-\alpha^s_t) = a_t$, after some manipulation, we get:

$$VSL_t = \frac{\beta}{1-\beta}\frac{1}{c_t^{-\sigma}}\frac{\left(-\frac{E_t\left[e^{-kV_{t+1}(b_t,A_t,s_t,\eta_t,\zeta_t)}\right] - E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]}{k}\right)}{\pi_t E_t\left[e^{-kV_{t+1}(b_t,A_t,s_t,\eta_t,\zeta_t)}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]} \tag{S.15}$$
$$- \beta a_t \frac{E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} e^{-kV_{t+1}(b_t,A_t,s_t,\eta_t,\zeta_t)}\right]}{\pi_t E_t\left[e^{-kV_{t+1}(b_t,A_t,s_t,\eta_t,\zeta_t)}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]},$$

and in the additive case, by continuity for $k \to 0$:

$$VSL^{add}_t = \frac{\beta}{c_t^{-\sigma}}\left(\frac{V^{add}_{t+1}(b_t, A_t, s_t, \eta_t, \zeta_t)}{1-\beta} - kv(x_{t+1})\right) - \beta a_t E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\right]. \tag{S.16}$$

S.3.4 The limit cases in the EZW model

We consider the no-bequest case. The recursion defining the utility V_t representing EZW preferences is:

$$V_t = \left((1-\beta)c_t^{1-\sigma} + \beta\pi_t^{\frac{1-\sigma}{1-\gamma}}\left(E_t[V_{t+1}^{1-\gamma}]\right)^{\frac{1-\sigma}{1-\gamma}}\right)^{\frac{1}{1-\sigma}}. \tag{S.17}$$

We consider the limits of the utility function for $\sigma \to 1$ and $\gamma \to 1$.

S.3.4.1 The limit $\sigma \to 1$, while $\gamma \neq 1$.

We conduct a first-order development to compute the limit when $\sigma \to 1$. Taking the log of (S.17), we deduce that preferences can be represented by $\tilde{V}_t = \log V_t$ defined by the following recursion:

$$\tilde{V}_t = (1-\beta)\log(c_t) + \frac{\beta}{1-\gamma}\log(\pi_t E_t[e^{(1-\gamma)\tilde{V}_{t+1}}]), \tag{S.18}$$

which corresponds to the risk-sensitive utility recursion with $k = \gamma - 1$ and $ku_d = \infty$. Note that recursion (S.18) requires $\pi_t > 0$ at all dates. Otherwise, if $\pi_{T_{\max}} = 0$ for some $T_{\max}$, then $(1-\gamma)\tilde{V}_t = -\infty$ for all $t \leq T_{\max}$.

S.3.4.2 The limit $\gamma \to 1$, while $\sigma \neq 1$.

Such limit case is either undefined, degenerate or exhibits implausible impatience pattern. To show that one may focus on the case where the only uncertainty is related to survival, so that the expectation sign E_t can be removed in (S.17). The recursion (S.17) then becomes:

$$V_t = \left((1-\beta)c_t^{1-\sigma} + \beta\pi_t^{\frac{1-\sigma}{1-\gamma}} V_{t+1}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

To discuss the limit, one has to distinguish the cases where γ tends to 1 from above or from below, and whether σ is larger or smaller than 1.

In the case where $\gamma \to 1^-$ and $\sigma < 1$ then $\frac{1-\sigma}{1-\gamma} \to +\infty$ and $V_t \to (1-\beta)^{\frac{1}{1-\sigma}} c_t$. The model then exhibit infinite impatience, where the future does not matter. The case where $\gamma \to 1^+$ and $\sigma > 1$ is similar. When $\gamma \to 1^+$ and $\sigma < 1$ then $\frac{1-\sigma}{1-\gamma} \to -\infty$ and $V_t \to \infty$, the limit being then undefined. Last, when $\gamma \to 1^-$ and $\sigma > 1$ then $\frac{1-\sigma}{1-\gamma} \to -\infty$ and $V_t \to 0$, the limit model being a degenerate one.

S.3.4.3 The limit $\gamma \to 1$ and $\sigma \to 1$.

Due to the issues that appear when taking the limit $\gamma \to 1$, the results is sensitive to the order in which the limits $\gamma \to 1$ and $\sigma \to 1$ are considered. However, the limit utility is always ill-defined, being equal to either 0 or infinity, independently of consumption choices.

S.4 Details on the computational implementation

There is no analytical solution to the agents' problem outlined in Section 4 of the main paper. We solve the model and obtain decision rules numerically and then use those decision rules to simulate the agents' behavior. The next two subsections describe the solution of the model and the simulation of decision rules.

S.4.1 Model solution

While alive, the agent maximizes her intertemporal utility by choosing a feasible allocation $(c_t, b_t, a_t, s_t, \eta_t)_{t\geq 0}$ in the set of feasible allocations, denoted $\mathcal{A}$. The utility V_t of the alive agent at age t depends on five state variables: the beginning-of-period holdings in bonds b_{t-1}, annuities A_{t-1} and stocks s_{t-1}; the stochastic component of labor income, ζ_{t-1}; and the stock market participation status, $\eta_{t-1} \in \{0, 1\}$. The last of these is discrete, while the first four are continuous. Given that annuity purchase may only occur in period $T_R - 1$, we have $A_t = 0$ for all $t < T_R$ and $A_t = a_{T_R-1}$ for $t \geq T_R$. Since there exists a maximal age for the agent, T_M, we solve the model by iterating on the value function, starting from the last period of life. Utility maximization involves solving:

$$V_{T_M}(b_{T_m-1}, A_{T_m-1}, s_{T_m-1}, \eta_{T_m-1}, \zeta_{T_m-1}) = \max_{(c_{T_M}, b_{T_M}, a_{T_M}, s_{T_M}, w_{T_M}, \eta_{T_M}) \in \mathcal{A}} (1-\beta)u(c_{T_m}) + E_t\left[e^{-k(1-\beta)v(x_{T_m+1})}\right], \tag{S.19}$$

subject to the constraints, which we don't restate here, outlined in Section 4 of the main paper. Due to the presence of stocks in bequest, the continuation utility in case of death is uncertain. Both the instantaneous utility function for period $T_m(u(x_{T_m}))$ and the utility obtained when dying and bequeathing $x_{T_{M+1}}(v(x_{T_{M+1}}))$ are known and the model is solved for a discrete set of points on a grid. This gives us knowledge of V_{T_m} at a subset of the points in the state space and allows us to approximate V_{T_m} as $\hat{V}_{T_m}$ at all points. With this approximation in hand, we solve an approximation to problem (S.19) for period $T_M - 1$ and then, iteratively, for all preceding periods t:

$$V_t(b_{t-1}, A_{t-1}, s_{t-1}, \eta_{t-1}, \zeta_{t-1}) = \max_{(c_t, b_t, a_t, s_t, w_t, \eta_t) \in \mathcal{A}} (1-\beta)u(c_t) - \frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-k\hat{V}_{t+1}(b_t, A_t, s_t, \eta_t, \zeta_t)}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]\right). \tag{S.20}$$

Note that the only difference between the maximand in (S.20) and that in our household's problem is that the continuation value function is an approximate value function. We will now, briefly, discuss four features of the numerical procedure: i) the discretization of the continuous variables, ii) the integration of the value function, iii) the approximation method for evaluating the value function at points outside the discretized state space and iv) how the optimization is carried out.

Discretization of the continuous variables. We define a variable, total liquid wealth, which is the sum of bond and stock holdings at the start of a period. We define a grid of 54 points from \$0 to \$10m such that the gaps between successive grid points are smaller at lower levels of wealth, where the curvature of the value function will be greater. We define a grid of 36 points for annuity income from \$0 to \$800,000 such that the gaps between successive grid points are smaller at lower levels of income. Earnings are placed on a grid of 9 points each year using the procedure introduced by Tauchen (1986).

Integration. There are three risks facing households: mortality, earnings, and financial risks. The risks are independent. Realizations for the first of these are naturally discrete and integration involves a simple weighted average. For the latter two, we define a discrete set of possible realizations and integrate over outcomes using the Tauchen (1986) procedure.

Approximation. To evaluate the value function at points other than those in the discrete sub-set of points, we use linear interpolation in multiple dimensions.

Optimization. Every period households make up to four choices. They decide how much to consume, how much to save in each of bonds and the risky asset, whether to pay the participation cost (if they have not previously done so) and how much of an annuity income stream to purchase (in the period before retirement). Our problem is not globally concave, so our optimization of the household's decision problem cannot fully rely on local approaches. We therefore start by discretizing the choice variables. We define three grids on the unit interval, S_a^t, S_s, S_c, which will represent shares of available resources dedicated to purchases of annuities, portfolio shares in stocks and consumption respectively. All grids have equally spaced nodes from 0 to 1 (except that in periods other than period T_{R-1}, the grid for annuities has only one element, 0, as no annuity purchase is possible in those periods). We evaluate the household's objective function at each combination of $(s_{a,i}^t)_{i=1,\dots,I^t}$ in S_a^t, $(s_{s,j})_{j=1,\dots,J}$ in S_s, and $(s_{c,k})_{k=1,\dots,K}$ in $S_{c,k}$. Defining the total available resources as $R_t = y_t + w_t$ (the sum of income and wealth) in time t, we set the annuity choice to $s^t_{a,i} R_t$, the level of saving in the risky asset set to $s_{s,j}(1 - s^t_{a,i})R_t$, the level of consumption as $s_{c,k}(1 - s_{s,j})(1 - s^t_{a,i})R_t$ and the savings in the bond as $(1 - s_{c,k})(1 - s_{s,j})(1 - s^t_{a,i})R_t$. We find the combination of the points that yields the maximum value to the household. This is our candidate optimal decision. We then do a further local search for the split between consumption and the bond using golden section search. Formally, taking the candidate maximum as indexed by s^t_{a,i^*} in S^t_a, s_{s,j^*} in S_s, s_{c,k^*} in $S_{c,k}$, we look for the utility-maximizing split between consumption and the bond in the interval for consumption of $[s_{c,\max\{1,k^*-1\}}(1-s_{s,j^*})(1-s^t_{a,i^*})R, s_{c,\min\{K,k^*+1\}}(1-s_{s,j^*})(1-s^t_{a,i^*})R]$.

For points where individuals have not already paid the participation charge, we implement this procedure twice, once assuming they pay the charge, once assuming they do not. The maximum of these indicates the decision rule.

This procedure yields decision rules as a function of the vector of state variables $\boldsymbol{X}_t$: $\hat{c}(\boldsymbol{X}_t)_t, \hat{b}(\boldsymbol{X}_t)_t, \hat{s}(\boldsymbol{X}_t)_t, \hat{a}(\boldsymbol{X}_t)_t, \hat{\eta}(\boldsymbol{X}_t)_t$.

S.4.2 Simulation of profiles

Once decision rules $\hat{c}(\boldsymbol{X}_t)_t, \hat{b}(\boldsymbol{X}_t)_t, \hat{s}(\boldsymbol{X}_t)_t, \hat{a}(\boldsymbol{X}_t)_t, \hat{\eta}(\boldsymbol{X}_t)_t$ are obtained, we simulate a data set for 3,000 individuals. We do this as follows:

1. Initial values for wealth are set to 0.

2. Earnings draws for the first period of economic life are drawn randomly for each individual. Using these values of the state variables and the decision rules we can obtain optimal behavior in the first period.

3. We draw an equity price shock to apply to any equity holdings held at the end of the period.

4. Optimal behavior, the rate of return and the inter-temporal budget constraint yield the state variables for period 2.

5. We repeat steps (2) to (4) to obtain optimal behavior and subsequent state variables for each age up to 100. In most time periods, individuals will have realizations of the continuous state variables that are off this grid. Our approach here is to solve the individual's problem for decision rules as we do in the solution stage.

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