# Recursive Preferences and the Value of Life. Theory and Application to Epidemics

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**Abstract**

This article investigates how recursive preferences can be used in the context of lifecycle models featuring uncertain and endogenous lifespans. We provide representation results showing how recursive preferences may be homothetic or fulfill a simple form of monotonicity with respect to first-order stochastic dominance – called ordinal dominance. While homotheticity appears to be very restrictive, constraining the intertemporal elasticity of substitution to be above one, ordinal dominance points to the risk-sensitive preferences of Hansen and Sargent (1995), on which we focus for the second part of the paper. We then discuss the theoretical impact of risk aversion, and illustrate the relevance of our findings by looking at the consumption-mortality trade-offs faced by a benevolent planner during a pandemic.

**Keywords**: value of life, recursive utility, lifecycle models, epidemics.

**JEL codes**: G11, J17.

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# 1 Introduction

Major epidemics like the plague, the Spanish flu or the ongoing Covid-19 pandemic may cause the death of millions of people. One solution to reduce the number of fatalities involves limiting human interactions, which however may imply a huge economic cost. Societal decision-making therefore has to consider a trade-off between survival probabilities and wealth, which requires an economic value be set (implicitly or explicitly) for mortality risk reductions. The literature that addresses this trade-off typically uses the seminal theoretical framework introduced by Yaari (1965), where agents' lifetime utility is the sum of instantaneous utilities weighted by survival probabilities. As is well-known, such additive models lack flexibility to disentangle risk aversion from intertemporal elasticity of substitution (IES, henceforth). Using additive models thus prevents proper accounting for risk aversion, which, however, is likely to be a major behavioral trait in the presence of mortality risk.

Following the works of Epstein and Zin (1989) and Weil (1989) (EZW, henceforth), recursive utility models have become the most popular tool to address the role of risk aversion in intertemporal contexts, with applications in numerous fields. While recursive preferences were initially developed to address long-standing puzzles in the macro-finance literature, they are now increasingly employed in other fields such as the economics of climate change, health economics, or household finance. In the current paper, we provide a theoretical investigation on how recursive models may help clarify the role of risk aversion when discussing the value of mortality risk reduction. We then highlight how accounting for risk aversion may provide new insights when considering wealth-survival trade-offs.

Our paper includes a theoretical part, with three contributions, and an application to epidemics. Our first theoretical contribution consists in two representation results. Our starting point are recursive preferences, that are monotone and do not systematically predict a negative value of life. Monotonicity means that an agent's utility increases with consumption, while the assumption of a non-systematically negative value of life requires that an agent prefers to be alive and to consume rather than to be dead, at least for some consumption levels. Our first representation result is that when we further impose Homotheticity to these well-behaved preferences, we obtain either standard additive preferences or (homothetic) EZW preferences, but where the IES is restricted to be greater than one and the coefficient of risk aversion smaller than one. When instead of Homotheticity, we impose Ordinal Dominance (similar to monotonicity with respect to first-order stochastic dominance), we obtain risk-sensitive preferences à la Hansen and Sargent (1995) – with no particular restriction on utility parameters. The only preferences simultaneously fulfilling Homotheticity and Ordinal Dominance are the standard additive preferences, which do not allow one to separate IES from risk aversion. A consequence of these representation results is that disentangling risk aversion from IES requires either Ordinal Dominance or Homotheticity be abandoned. On the one hand, opting for Homotheticity and giving up Ordinal Dominance has three main drawbacks: (i) the agent may end up opting for dominated choices (see Bommier et al., 2017 or Bommier et al., 2020 for illustrations), (ii) the IES and risk aversion parameters are constrained in a way that is no consistent with empirical evidence, and (iii) it involves assuming that rich and poor behave identically (up to a wealth scaling factor) which is also inconsistent with empirical evidence. On the other hand, opting for Ordinal Dominance and discarding Homotheticity mostly slightly deters model tractability, but preserves model insights and does not constrain model parameters. Our take-away of these representation results is that imposing Homotheticity for tractability reasons is extremely costly. Meanwhile, the risk-sensitive framework provides an appealing approach to discuss value of life matters, with sufficient flexibility to account for risk aversion.

Our second theoretical contribution is to explain why our results regarding EZW specifications differ from the ones in the literature, and especially from those of Córdoba and Ripoll (2017) – hereafter CR – and Hugonnier et al. (2013) – hereafter HPSA.[1] These papers argued that homothetic EZW specifications could cope with

[1]These papers, which were developed independently provide very similar modelling approaches,

an IES smaller than one. We explain that this actually results from a mathematical error in CR, and from the assumption of an unrealistic preference domain in HPSA.

Our third theoretical contribution involves showing that working with risk-sensitive preferences allows one to derive results regarding the discount rate and the value of mortality risk reduction. Regarding the discount rate, we show that agents become more impatient (and have a lower discount rate) when: (i) they are more risk averse, (ii) they face a higher mortality probability, and (iii) their continuation utility is higher. The value of mortality risk reduction is also impacted by risk aversion and continuous utility. From a purely theoretical point of view, the impact of risk aversion is ambiguous, reflecting general results on the relation between optimal prevention and risk aversion (see, e.g., Jullien et al., 1999). The impact of continuation utility is unambiguously positive. Numerical illustrations using a realistic mortality pattern complement these findings, providing a quantitatively clear picture: risk aversion tends to increase the value of mortality risk reduction, and also changes the relation between age and the value of mortality risk reduction. The explanation for this latter aspect is that risk aversion amplifies the willingness to avoid particular adverse outcomes (such as death at a young age), compared to the willingness to avoid less adverse outcomes (such as death at an advanced age).

In order to illustrate the relevance of using a recursive specification, we focus on two real-world cases, namely the Covid-19 and 1918 influenza pandemics, and demonstrate how recursive preferences alter the conclusions regarding the optimal consumption-mortality trade-off. Our results highlight that the sign of corrections depends on whether the pandemic predominantly impacts older people (as with Covid-19) or younger people (as with the 1918 influenza outbreak). For Covid-19, accounting for risk aversion through recursive preferences would tend to reduce the amount of consumption that the social planner would be willing to sacrifice in order

based on EZW preferences. The main difference is that HPSA uses a continuous-time setting, which makes the mathematics more complex, while CR assumes that time is discrete. Both papers have been used in several follow-up articles, which suffer from the same shortcomings as the ones we detail in the current contribution.

to limit mortality. Concretely, this would argue in favor of a quicker reopening of the economy, or a less severe lock-down, than what is advised when using usual additive preferences. For the 1918 influenza, the conclusions are inverse. The reason for those findings is that models are usually calibrated upon observations of wage-risk trade-offs made by workers (thus of “middle-aged” people), while accounting for the mortality impact of pandemics requires us to infer the value of mortality risk reduction at younger and older ages. A form of extrapolation is thus needed, which is typically achieved through the use of a specific model of individual preferences. The structure of the model adopted ends up playing a key role in this extrapolation. In particular, in comparison to additive models, recursive models end up giving greater weight to particularly adverse consequences (i.e., death of younger people) and less weight to less dramatic consequences (i.e., death of older people). This of course reflects the very natural role of risk aversion, which could not be properly investigated with the standard additive model.

# 2 Studying the value of life with recursive models

## 2.1 Temporal lotteries with uncertain lifetime

Recursive models were first suggested to model preferences over temporal lotteries in fixed or infinite horizon settings. To model choices under uncertain lifespans, we need to consider lives of unequal lengths. For the sake of simplicity we will assume that there is a finite upper bound $T_{max}$ on how long a life can be.[2] We will use the letter

[2]Since $T_{max}$ can be arbitrarily large, there is no significant loss of generality. Moreover, the assumption of a finite lifespan is consistent with demographic evidence. Jeanne Calment is reported to have the longest lifespan of 122 years and 164 days and is the only human to have lived beyond the age of 120 years. Maximal biological age is also supported by biological evidence (Weon and Je, 2009; Dong et al., 2016).

$d$ to describe the death state and define the set of temporal lotteries as follows:[3]

$$\begin{cases} D_t = \{d\} & \text{for } t = T_{max}, \\ D_t = (\mathbb{R}_+ \times M(D_{t+1})) \cup \{d\} & \text{for } t \in \{0, \ldots, T_{max} - 1\}, \end{cases}$$

where for any set $X$ the notation $M(X)$ denotes the set of simple lotteries with outcomes in $X$. As is usual for any element $x \in X$ we shall use the same notation $x$ to denote the degenerate lottery in $M(X)$ which gives $x$ with probability one.

An element of $D_t$ different from $d$ (thus reflecting a situation where the agent is alive in period $t$) will be typically denoted by a pair $(c_t, m_t)$ where $c_t \in \mathbb{R}_+$ is the consumption in period $t$ and $m_t \in M(D_{t+1})$ is a lottery over future states. For any $m_t \in M(D_{t+1})$, we define the survival probability $\pi(m_t)$, by $\pi(m_t) = 1 - Prob_{m_t}(d)$, where $Prob_{m_t}(d)$ is the occurrence probability of being dead in period $t + 1$. When $\pi(m_t) \neq 0$, we will also define $m_t^S \in M(D_{t+1} \setminus \{d\})$ by:

$$m_t = \pi(m_t) m_t^S \oplus (1 - \pi_t(m_t))d, \tag{1}$$

where $\oplus$ denotes the standard mixture operation over lotteries. The above equation thus simply states that $m_t$ can be seen as a mixture, with weights $(1 - \pi(m_t))$ and $\pi(m_t)$, of a lottery that gives the death state for sure and a lottery $m_t^S$ whose outcomes exclude (immediate) death. For an element $(c_t, m_t) \in D_t \setminus \{d\}$, which describes the case of an agent alive in period $t$, the probability $\pi(m_t)$ is the probability of staying alive from period $t$ to period $t + 1$, and $m_t^S$ is the lottery describing the distribution of outcomes in period $t + 1$ conditional on being alive.

We (recursively) define a "multiplication by a scalar operation" over the spaces $D_t$ as follows:

$$\begin{cases} \lambda d = d & \text{for all } \lambda \in \mathbb{R}_+, \\ \lambda(c_t, m) = (\lambda c_t, \lambda m) & \text{for all } \lambda \in \mathbb{R}_+ \text{ and } (c_t, m) \in M(D_t \setminus \{d\}), \end{cases} \tag{2}$$

[3] The death state $d$ is neither a consumption level nor a utility level. It has no other interpretation but materializing the terminal leaf of temporal lotteries.

Thus, multiplying a temporal lottery by $\lambda$ involves multiplying all (current and future) consumption levels by $\lambda$, with no impact on the survival probabilities.

As an example, the element

$$(c_1, m_1) = (c_1, \frac{1}{3}(c_2, d) \oplus \frac{1}{3}(c_2', d) \oplus \frac{1}{3}d) \tag{3}$$

describes the case of an agent who is alive and consumes $c_1$ in period 1 and then dies with probability $\frac{1}{3}$ or survives with probability $\pi(m_1) = \frac{2}{3}$. If surviving in period 2, she consumes either $c_2$ or $c_2'$ with equal probabilities, and then dies for sure at the end of period 2 (formally $m_1^S = \frac{1}{2}(c_2, d) \oplus \frac{1}{2}(c_2', d)$). In such a case, for any $\lambda \in \mathbb{R}_+$ one has:

$$\lambda(c_1, m_1) = (\lambda c_1, \frac{1}{3}(\lambda c_2, d) \oplus \frac{1}{3}(\lambda c_2', d) \oplus \frac{1}{3}d).$$

A graphical representation of the temporal lottery (3) is provided in Figure 1.

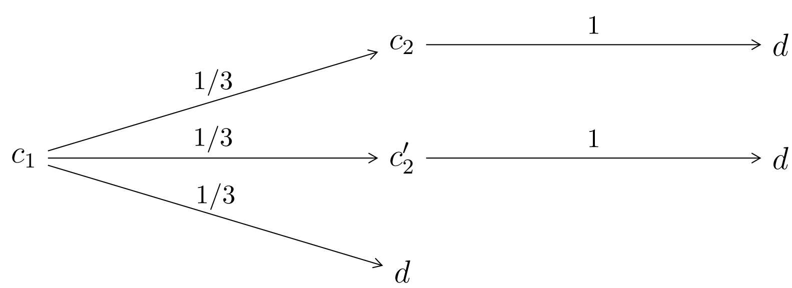


Figure 1: A graphical representation of the consumption lottery $(c_1, \frac{1}{3}(c_2, d) \oplus \frac{1}{3}(c_2', d) \oplus \frac{1}{3}d)$.

## 2.2 Well-behaved recursive preferences

In what follows, the agent is assumed to have preferences over the sets $D_t$, for $t \in \{0, \ldots, T_{max}\}$, represented by utility functions $U_t : D_t \to Im(U_t) \subset \overline{\mathbb{R}}$ related through the following recursion:

$$\begin{cases} U_t(d) = u_d, \\ U_t(c_t, m_t) = u(c_t) + \beta \phi^{-1}(E_{m_t}[\phi(U_{t+1})]), \end{cases} \tag{4}$$

where $u_d \in \overline{\mathbb{R}}$ is the utility associated with death, $u : \mathbb{R}_+ \to \overline{\mathbb{R}}$ is the period utility of consumption, $\phi$ is an increasing function defined over the convex hull of $\cup_{t \in \{0,\ldots,T_{max}\}} Im(U_t)$ and $\beta > 0$ is the discount factor.[4] Since $D_{Tmax} = \{d\}$, for a given combination of $u_d$,$u$, $\beta$ and $\phi$, there exists a unique sequence of functions $U_t$ that fulfills (4). Starting from $t = T_{max}$ and applying the recursion (4) backward provides a direct construction of the utilities $U_t$.

The functions $u$ and $\phi$ will be assumed to be twice continuously differentiable. The function $u$ governs intertemporal substitutability of consumption and will be assumed to be concave. It will be said to be CRRA (for Constant Relative Risk Aversion) if $u(c) = \frac{c^{1-\sigma}}{1-\sigma} + u_l$, with $0 < \sigma \neq 1$, or $u(c) = \ln(c) + u_l$, for some constant $u_l \in \mathbb{R}$. The function $\phi$ governs risk aversion, with greater concavity reflecting greater risk aversion. As is well known, such recursive preferences may exhibit preference for the timing of resolution of uncertainty. Precisely, it follows from Kreps and Porteus (1978, Theorem 3) that preferences exhibit preference for early (resp. late) resolution if the function $x \to \phi(u(c) + \beta\phi^{-1}(x))$ is convex (resp. concave). Recursive preferences also contrast with the standard additive framework for the possibility of exhibiting intertemporal correlation aversion. A formal definition of intertemporal correlation aversion can be found in Stanca (2021), who explains that a positive intertemporal correlation aversion is obtained whenever the function $\phi$ is concave.

Using survival probabilities and the mixture operation of equation (1), recursion (4) can be rewritten as $U_t(c_t, m_t) = u(c_t) + \beta\phi^{-1}(\pi_t(m_t)E_{m_t^S}[\phi(U_{t+1})] + (1 - \pi_t(m_t))\phi(u_d))$. For greater legibility, we will simplify the notation in the remaining and simply write:

$$U_t(c_t, m_t) = u(c_t) + \beta\phi^{-1}\left(\pi_t E[\phi(U_{t+1})] + (1 - \pi_t)\phi(u_d)\right), \tag{5}$$

where $\pi_t$ implicitly stands for the probability $\pi_{(m_t)}$ and $E[\cdot]$ for the expectation

[4]Note that there is no requirement that $u_d$ belongs to the image of the instantaneous utility function $u$. In particular there does not need to exist a "death consumption equivalent" (that is a consumption level $c_d$ such that $u(c_d) = u_d$). The only actual requirement is that $\phi$ is well-defined on the convex hull of $\cup_t Im(U_t)$.

operator $E_{m_t^S}[\cdot]$.

Throughout the paper, we will assume that preferences fulfill two basic requirements, stipulating that greater consumption implies greater welfare and that at least some lives are considered as worthwhile (i.e., better than death). Formally:

**Definition 1 (Well-behaved preferences)** *Recursive preferences with uncertain lifetime represented as in (4) are said to be well-behaved if:*

1. ***(Monotonicity)*** *The function $(c_0, c_1) \mapsto U_1(c_0, \frac{1}{2}(c_1, d) \oplus \frac{1}{2}d)$ is strictly increasing.*

2. ***(Non-Systematically Suicidal)*** *For all $t < T_{max}$, there exists $(c_t, m) \in D_t$ such that $U_t(c_t, m) > u_d$.*

The assumptions "Monotonicity" and "Non-Systematically Suicidal", which we view as basic requirements, allow specific normalization to be made, thus simplifying the utility representation. Indeed:

**Proposition 1** *Recursive preferences with uncertain lifetime are well-behaved if and only if they admit a utility representation $U_t$ where:*

- *the period utility function $u$ is strictly increasing, and not always negative (i.e., there exists $c \in \mathbb{R}_+$ such that $u(c) > 0$);*

- *we can set the normalization: $u_d = \phi(u_d) = 0$.*

*The utility $U_t$ can then be defined through the following recursion:*

$$\begin{cases} U_t(d) = 0, \\ U_t(c_t, m) = u(c_t) + \beta\phi^{-1}(\pi_t E[\phi(U_{t+1})]), \end{cases} \tag{6}$$

*with: $\phi(0) = 0$ and $u' > 0$.*

For the remainder of the paper we will focus on well-behaved recursive preferences represented as in (6), and explore the additional restrictions that would be related to two additional assumptions, namely "Homotheticity" and "Ordinal Dominance".

## 2.3 Homotheticity and Ordinal Dominance

Preference homotheticity means that scaling all present and future consumption levels by the same (positive) factor does not change the ranking of lotteries. This is formalized in the following axiom:

**Axiom 1 (Homotheticity)** *For any* $t \geq 0$*,* $(c_t, m), (c'_t, m') \in D_t$ *and* $\lambda > 0$ :

$$(U_t(c_t, m) \geq U_t(c'_t, m')) \Leftrightarrow (U_t(\lambda c_t, \lambda m) \geq U_t(\lambda c'_t, \lambda m')) \, ,$$

Homotheticity is a very popular assumption as it tends to simplify optimization problems. Indeed, under preference homotheticity, wealth has a basic scaling effect and can be easily taken out of optimization problems. This is very convenient, especially in settings where wealth is impacted by some random factors (as asset returns, or random labor income) along the lifecycle.

**Proposition 2** *Well-behaved recursive preferences with uncertain lifetime fulfill Homotheticity (Axiom 1) iff they admit a utility representation* $U_t$ *fulfilling recursion (6) in which:*

- *either* $u$ *is CRRA, not always negative, and* $\phi$ *is linear,*
- *or* $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ *for some* $\sigma < 1$*, and* $\phi(x) = x^\rho$ *for some* $\rho > 0$*. The parameter* $\rho$ *governs risk aversion (with a larger* $\rho$ *implying a lower degree of risk aversion), while* $\frac{1}{\sigma}$ *is the intertemporal elasticity of substitution*

*Formally, utility* $U_t$ *can then be defined by* $U_t(d) = 0$ *and one of the following recursions:*

- $U_t(c_t, m) = u(c_t) + \beta \pi_t E[U_{t+1}]$ *with* $u$ *CRRA, not always negative;*
- $U_t(c_t, m) = \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \left(\pi_t E[U_{t+1}^\rho]\right)^{\frac{1}{\rho}}$ *with* $\sigma < 1$ *and* $\rho > 0$.

The first case corresponds to the standard additive specification, which lacks flexibility to study the role of risk aversion. The second case corresponds to Epstein-Zin-Weil preferences, with an IES, $\frac{1}{\sigma}$, above one and a coefficient of relative risk aversion, $1-\rho(1-\sigma)$, smaller than one. We will explain in Section 2.4 below why the above results contrast with the messages found in CR and HPSA.

The other axiom we consider is Ordinal Dominance ("OD", thereafter).

**Axiom 2 (Ordinal Dominance)** *For all dates* $0 \leq t < T_{max}$*, consumption levels* $c_t, c_t' \in \mathbb{R}_+$ *and lotteries* $m_1, m_1', m_2, m_2' \in M(D_{t+1})$*, if:*

$$\begin{cases} U_t(c_t, m_1) \geq U_t(c_t', m_1'), \\ U_t(c_t, m_2) \geq U_t(c_t', m_2'), \end{cases}$$

*then:*

$$U_t(c_t, \frac{1}{2}m_1 \oplus \frac{1}{2}m_2) \geq U_t(c_t', \frac{1}{2}m_1' \oplus \frac{1}{2}m_2').$$

OD is defined in very similar fashion in Chew and Epstein (1990), or in Bommier et al. (2017) for temporal lotteries, and adapted here to the context of uncertain lifetime. It states that if lottery $(c_t, m_1)$ is preferred to $(c_t', m_1')$ and $(c_t, m_2)$ to $(c_t', m_2')$, then the mixture of the two most preferred should also be preferred to the mixture of the two least preferred. OD is similar in spirit to a property of preference monotonicity with respect to first-order stochastic dominance. Note that Axiom 2 only requires a mixture with a 50%-50% probability. However, in our setup, preferences that fulfill Axiom 2 will also fulfill a stronger version with arbitrary probabilities.

**Proposition 3** *Well-behaved recursive preferences with uncertain lifetime fulfill OD (Axiom 2) if and only if they admit a utility representation* $U_t$ *fulfilling recursion (6) in which* $\phi(x) = \frac{1-e^{-kx}}{k}$ *for some* $k \neq 0$ *or* $\phi(x) = x$ *and the function* $u$ *is strictly increasing and not always negative. Utility* $U_t$ *is then defined by* $U_t(d) = 0$ *and the* *following recursion:*

$$U_t(c_t, m) = \begin{cases} u(c_t) - \frac{\beta}{k} \ln\left(\pi_t E\left[e^{-kU_{t+1}}\right] + 1 - \pi_t\right) & \text{for } k \neq 0, \\ u(c_t) + \beta \pi_t E[U_{t+1}] & \text{for } k = 0. \end{cases} \tag{7}$$

Specification (7) is nothing other than an adaptation of risk-sensitive preferences to a context of an uncertain lifetime. Indeed, Proposition 3 can be seen as an extension of the representation result of Bommier et al. (2017) to uncertain horizons but restricted to expected utility certainty equivalents, as in Kreps and Porteus (1978).[5] From Bommier and LeGrand (2014), we also know that the preferences represented by recursion (7) verify a general notion of monotonicity with respect to first-order stochastic dominance that encompasses our definition of OD. With such preferences, if a consumption choice dominates (in the sense of first-order stochastic dominance) another consumption choice, the first choice will also be preferred to the second one. If one views taking decisions under uncertainty as playing a game against Nature, OD is akin to the elimination of dominated strategies. It thus appears to be a very natural assumption to model rational choice.

A consequence of Propositions 2 and 3 is that the only preferences fulfilling Homotheticity and OD are the standard additive preferences. This is formulated in the following corollary.

**Corollary 1** *Well-behaved recursive preferences with uncertain lifetime fulfill Homotheticity (Axiom 1) and OD (Axiom 2) and if and only if they can be represented as in (6) with a function $u$ that is CRRA and not always negative, and $\phi(x) = x$.*

It follows from Corollary 1 that disentangling IES from risk aversion requires Homotheticity or OD to be abandoned. On the one hand, giving up Homotheticity and choosing OD leads to preferences that are overall well-behaved, even though

[5] The assumption of expected utility certainty equivalents is undoubtedly restrictive and has been challenged by experimental evidence (Camerer and Ho, 1994). It seems however to us to be a good starting point to explore implications of passing from risk-neutral certainty equivalents (as in the standard additive framework) to risk-averse certainty equivalents.

they come with a slight tractability burden. On the other hand, giving up OD and choosing Homotheticity yields preferences that may end up opting for dominated choices. Second, the IES is restricted to be above one, and risk aversion below one which are both restrictive assumptions in contradiction with empirical evidence. Finally, while the Homotheticity assumption is technically convenient, its empirical relevance has regularly been challenged. Indeed, it is well-documented that rich and poor people do not behave identically (in relative terms) when makings choices such as savings and financial investments, for example, in direct contradiction with the assumption of preference homotheticity (see for instance Dynan et al., 2004 for saving behaviors and Calvet and Sodini, 2014 for portfolio compositions).

Overall, it seems to us that the overall balance is clearly in favor of OD and that the cost of maintaining Homotheticity is too high. From Proposition 2, this leads to opting for risk-sensitive preferences. We will explore in Section 2.7 the consequences of working with such risk-sensitive preferences, but we first clarify the relationship with the previous literature.

## 2.4 Working with EZW representations?

CR and HPSA, both published in the *Review of Economic Studies*, suggested working with homothetic EZW recursive preferences to discuss value of life matters. They claimed that these homothetic preferences disentangle risk aversion from intertemporal elasticity of substitution and could accommodate positive values of life without constraining the IES to be above one. Although these papers slightly differ in their setting – since HPSA use continuous-time, while CR work in discrete time – they share a number of common features. For our discussion, we will focus on the discrete-time setting of CR although we will briefly discuss some specific aspects of HPSA.[6]

EZW preferences are generally defined in infinite-horizon settings, where the

[6] A more extensive discussion of HPSA, using their continuous-time setting, can be found in the working paper version, Bommier et al. (2020).

utility $V_t$ representing EZW preferences is defined through the following recursion:

$$V_t = \left( (1-\beta)c_t^{1-\sigma} + \beta \left( E[V_{t+1}^{1-\gamma}] \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}}. \tag{8}$$

The parameters $\gamma$ and $\sigma$ are positive numbers reflecting risk aversion and the inverse of the intertemporal elasticity of substitution, respectively. The cases where $\sigma$ or $\gamma$ equal 1 can be obtained by continuity from the above formula.

Accounting for the possibility of death can be made by using the domain described in Section 2 and assuming a utility level to death that we denote by $v_d$. Formally, recursion (8) then becomes:

$$V_t = \left( (1-\beta)c_t^{1-\sigma} + \beta \left( \pi_t E[V_{t+1}^{1-\gamma}] + (1-\pi_t)v_d^{1-\gamma} \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}}. \tag{9}$$

This expression, which is also the one used in CR (equation 2 in their paper), looks different from that of equation (5). However, this is actually only a question of utility normalization. For example, when $\sigma \neq 1$ and $\gamma \neq 1$, one can then set $U_t = \frac{V_t^{1-\sigma}}{(1-\beta)(1-\sigma)}$ and $u_d = \frac{v_d^{1-\sigma}}{(1-\beta)(1-\sigma)}$, representing the same preferences as $V_t$ (remember that utility representation is defined up to an increasing transformation), and obtain a utility representation fulfilling the recursion (5). A similar transformation exists in the other cases where $\gamma$ or $\sigma$ is equal to 1. Formally, representation (9) is equivalent to representation (5) when using the correspondences of Table 1.

| Parameters | utility $U_t$ | utility $u(c)$ | death utility $u_d$ | Risk function $\phi(x)$ |
|---|---|---|---|---|
| $\sigma \neq 1$ and $\gamma \neq 1$ | $\frac{V_t^{1-\sigma}}{(1-\beta)(1-\sigma)}$ | $\frac{c^{1-\sigma}}{1-\sigma}$ | $\frac{v_d^{1-\sigma}}{(1-\beta)(1-\sigma)}$ | $\frac{1}{1-\gamma}\left((1-\sigma)(1-\beta)x\right)^{\frac{1-\gamma}{1-\sigma}}$ |
| $\sigma = 1$ and $\gamma \neq 1$ | $\frac{\ln(V_t)}{1-\beta}$ | $\ln(c)$ | $\frac{\ln(v_d)}{1-\beta}$ | $\frac{1}{1-\gamma}\exp\left((1-\gamma)(1-\beta)x\right)$ |
| $\sigma \neq 1$ and $\gamma = 1$ | $\frac{V_t^{1-\sigma}}{(1-\beta)(1-\sigma)}$ | $\frac{c^{1-\sigma}}{1-\sigma}$ | $\frac{v_d^{1-\sigma}}{(1-\beta)(1-\sigma)}$ | $\frac{1}{1-\sigma}\ln\left((1-\sigma)(1-\beta)x\right)$ |
| $\sigma = 1$ and $\gamma = 1$ | $\frac{\ln(V)}{1-\beta}$ | $\ln(c)$ | $\frac{\ln(v_d)}{1-\beta}$ | $x$ |

Table 1: Correspondences between representations (5) and (9).

Changing utility normalization is of course harmless and one may indifferently use representation (5) or (9), with the appropriate correspondence, to discuss fundamental preference properties. In particular, it directly follows from Lemmas 1 and 2, in

Appendix A.1, that when $v_d > 0$, the recursion (9) represents well-behaved preferences in the sense of Definition 1 (i.e., preferences that are monotone and NSS). Such specifications are for instance used in Pashchenko and Porapakkarm (2022). They are not homothetic (unless $\sigma = \gamma$, providing the additive model) and do not fulfill OD (unless $\sigma = \gamma$ or $\sigma = 1$).[7] Imposing Homotheticity (together with Monotonicity and NSS) is only possible if $\gamma = \sigma$ (additive case) or if $\sigma < 1$ and $\gamma < 1$ and $v_d = 0$. This latter case imposes stringent restrictions on the IES, $\frac{1}{\sigma}$, assumed to be above one, and risk aversion, assumed to be below one. We thus need to clarify why our message strongly contrasts with those of CR and HPSA, who both claim that homothetic EZW specifications with $\gamma < 1$ are well-behaved even when $\sigma \geq 1$. In the case of CR, we will see in Section 2.5 that this is due to a mathematical error in their analysis. Regarding HPSA, we will explain in Section 2.6 that this relates to a restriction on mortality rates, assumed to be bounded from above in a way that rules out the application to any realistic mortality pattern.

## 2.5 Mathematical error in Córdoba and Ripoll (2017)

CR claim that an EZW specification featuring $\gamma < 1$, $\sigma \geq 1$ and $v_d = 0$ could unproblematically accommodate positive value of life and homotheticity. When setting $v_d = 0$ and assuming $\gamma < 1$ and $\sigma > 1$, equation (9) becomes:

$$\begin{cases} V_t = \left((1-\beta)c_t^{1-\sigma} + \beta \pi_t^{\frac{1-\sigma}{1-\gamma}} E[V_{t+1}^{1-\gamma}]^{\frac{1-\sigma}{1-\gamma}}\right)^{\frac{1}{1-\sigma}} & \text{if } \sigma > 1, \\ V_t = c_t^{1-\beta} \pi_t^{\frac{\beta}{1-\gamma}} E[V_{t+1}^{1-\gamma}]^{\frac{\beta}{1-\gamma}} & \text{if } \sigma = 1. \end{cases} \tag{10}$$

However, we can infer from (10) that, in both cases, we have the following chain of implications:

$$V_{T_{max}} = 0 \Rightarrow V_{T_{max}-1} = 0 \Rightarrow V_{T_{max}-2} = 0 \Rightarrow \cdots \Rightarrow V_0 = 0.$$

[7] When the IES is equal to 1 ($\sigma = 1$), the non-homothetic EZW specification is also a risk-sensitive one, as is well-know from Tallarini (2000).

Such models are thus degenerate in the sense that they imply a zero utility at all dates, independently of consumption choices: $V_t = 0$ for all $t$ and all $c_t$.[8]

CR nevertheless used recursion (10) to derive first-order conditions (FOC hereafter) in optimization problems without checking whether utility is well-defined and non-degenerate. Most of their analysis relies on such first-order conditions. Section 5 of CR seems to provide a justification. In that section, CR consider the cases where $v_d > 0$ (which, as we explained, correspond to well-behaved models) and then discuss the limit where $v_d \to 0$. They conclude that the limit of such well-defined models leads to the same results as direct derivations of FOC from recursion (10). This suggests that the degeneracy issue mentioned above may be simply seen as a purely technical aspect that can be disregarded. The problem is that CR made an unfortunate mistake when handling the limit $v_d \to 0$. In fact, when $\sigma > 1$, such a limit involves an indeterminate form and is not adequately handled in CR. When this limit is computed correctly, one obtains FOC that are different from those derived from (10) and that have radically different predictions for agents' behavior.

To make the issue fully explicit, let us focus on the case where $\sigma > 1$ and mortality is the only source of uncertainty – such that the expectation symbol in (9) is no longer needed. Straightforward calculations provide the following marginal rate of substitution between consumption in period $t+1$ and period $t$:

$$\frac{\frac{\partial V_t}{\partial c_{t+1}}}{\frac{\partial V_t}{\partial c_t}} = \beta \pi_t \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \left(\pi_t + (1-\pi_t)\left(\frac{v_d}{V_{t+1}}\right)^{1-\gamma}\right)^{\frac{\gamma-\sigma}{1-\gamma}}. \tag{11}$$

The error in CR involves assuming that when $v_d \to 0$, then $\frac{v_d}{V_{t+1}} \to 0$ and therefore

[8] In an early working paper version (Bommier et al., 2020) we show that this result extends to the case where there is no maximal life duration, but where survival probabilities converge to zero as age tends towards infinity.

that:[9]

$$\frac{\frac{\partial V_t}{\partial c_{t+1}}}{\frac{\partial V_t}{\partial c_t}} \to \beta \pi_t^{\frac{1-\sigma}{1-\gamma}} \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma}. \tag{12}$$

The expression $\beta \pi_t^{\frac{1-\sigma}{1-\gamma}} \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma}$ mentioned above is also the marginal rate of substitution that is computed from recursion (10) when ignoring issues related to indeterminate form and the one used by CR in their consumption-saving problems.

The computation of the limit in CR is mistaken because when $v_d \to 0$, we also have $V_{t+1} \to 0$. Remember indeed that, as can be seen from recursion (9), $V_{t+1}$ depends on $v_d$. The limit $\lim_{v_d \to 0} \frac{v_d}{V_{t+1}}$ is thus of the indeterminate form $\frac{0}{0}$ (and in fact different from zero). In order to compute the limit correctly, one may rewrite recursion (9) as follows:

$$\frac{V_t}{v_d} = \left[ (1-\beta) \left( \frac{c_t}{v_d} \right)^{1-\sigma} + \beta \left( \pi_t \left( \frac{V_{t+1}}{v_d} \right)^{1-\gamma} + 1 - \pi_t \right)^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}}. \tag{13}$$

As $\sigma > 1$ implies that $\lim_{v_d \to 0} \left( \frac{c_t}{v_d} \right)^{1-\sigma} = 0$, we deduce that $\lim_{v_d \to 0} \frac{V_t}{v_d} = \chi_t$ where $(\chi_t)_{t \geq 0}$ is defined by $\pi_{T_{max}} = 0$ and

$$\chi_t = \beta^{\frac{1}{1-\sigma}} \left( \pi_t \chi_{t+1}^{1-\gamma} + 1 - \pi_t \right)^{\frac{1}{1-\gamma}} \text{ for } t = 0, \ldots, T_{max} - 1. \tag{14}$$

We therefore deduce from (11) that when $v_d \to 0$ ::

$$\frac{\frac{\partial V_t}{\partial c_{t+1}}}{\frac{\partial V_t}{\partial c_t}} \to \beta \pi_t \left( \pi_t + (1-\pi_t) \chi_{t+1}^{\gamma-1} \right)^{\frac{\gamma-\sigma}{1-\gamma}} \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma}, \tag{15}$$

which differ from the limit (12) computed by CR.

Considering the correct limit shown in equation (15), or the incorrect one written in equation (12), has a major impact on model predictions. To illustrate this, consider

[9] For precise referencing, the error in CR occurs at the beginning of their Section 5.1 (page 1503) where they write (with their notation): “The Euler equation in this case reads

$$c_{t+1}^{\sigma} = \beta(1+r) \left[ \pi + (1-\pi) \left( \underline{V}/V_{t+1} \right)^{1-\gamma} \right]^{\frac{\gamma-\sigma}{1-\gamma}} c_t^{\sigma}$$

which reduces to (23) [that is $c_{t+1}^{\sigma} = \beta(1+r)\pi^{\frac{\gamma-\sigma}{1-\gamma}} c_t^{\sigma}$] when $\underline{V} = 0$”. The assertion is false, as we explain in what follows (with the $\underline{V}$ of CR corresponding to $v_d$ in our contribution).

for example a standard lifecycle consumption-saving problem where an agent starts with some initial wealth $w_0$, and decides in each period how much to consume and how much to save. Assuming that the return on savings is $1+r$, the agent program can be expressed as follows:

$$V_t(w_t) = \max_{c_t, w_{t+1}} \left( (1-\beta) c_t^{1-\sigma} + \beta \left( \pi_t \left( V_{t+1}(w_{t+1}) \right)^{1-\gamma} + (1-\pi_t) v_d^{1-\gamma} \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}}, \quad (16)$$

$$\text{s.t. } w_t = c_t + \frac{1}{1+r} w_{t+1}. \quad (17)$$

The first-order condition of this problem is that the marginal rate of substitution $\frac{\partial V_t}{\partial c_{t+1}} / \frac{\partial V_t}{\partial c_t}$ between $c_{t+1}$ and $c_t$ should equal $\frac{1}{1+r}$. The model predictions regarding consumption-saving behavior when $v_d \to 0$ will thus depend on the limit for the marginal rate of substitution.

When choosing the wrong limit of equation (12), as in CR, the FOC would yield (when $v_d \to 0$):

$$\frac{c_{t+1}}{c_t} \to \left( \beta (1+r) \pi_t^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{\sigma}}, \quad (18)$$

which would mean that if $\sigma > 1 > \gamma$, mortality would reduce agents' impatience instead of contributing to it.[10] This would suggest that consumption and survival would be substitutes, which is hardly understandable in a setting where there is no bequest (and thus no utility for consumption after death).

Considering, instead, the correct limit of equation (15) one obtains, when $v_d \to 0$:

$$\frac{c_{t+1}}{c_t} \to \left( \beta (1+r) \pi_t \left( \pi_t + (1-\pi_t) \chi_{t+1}^{\gamma-1} \right)^{\frac{\gamma-\sigma}{1-\gamma}} \right)^{\frac{1}{\sigma}}, \quad (19)$$

which offers drastically different implications on the agent's consumption-saving behavior. To illustrate these differences in consumption-saving behavior, we compute consumption profiles implied by either (18) or (19) using benchmark parameters and realistic mortality profiles. The interest rate is assumed to be $r = 4\%$. Preference parameters are set to standard values (and have to respect $\gamma < 1$): $\beta = 0.98$, $\sigma = 2.0$,

[10] In the absence of annuities, survival probabilities have no impact on the budget constraint. The impact of survival probabilities on the optimal consumption profile is then a pure impatience effect.

and $\gamma = 0.5$. Mortality rates are those of the total US population in 2018, as reported in the Human Mortality Database (HMD). The first two consumption paths plotted

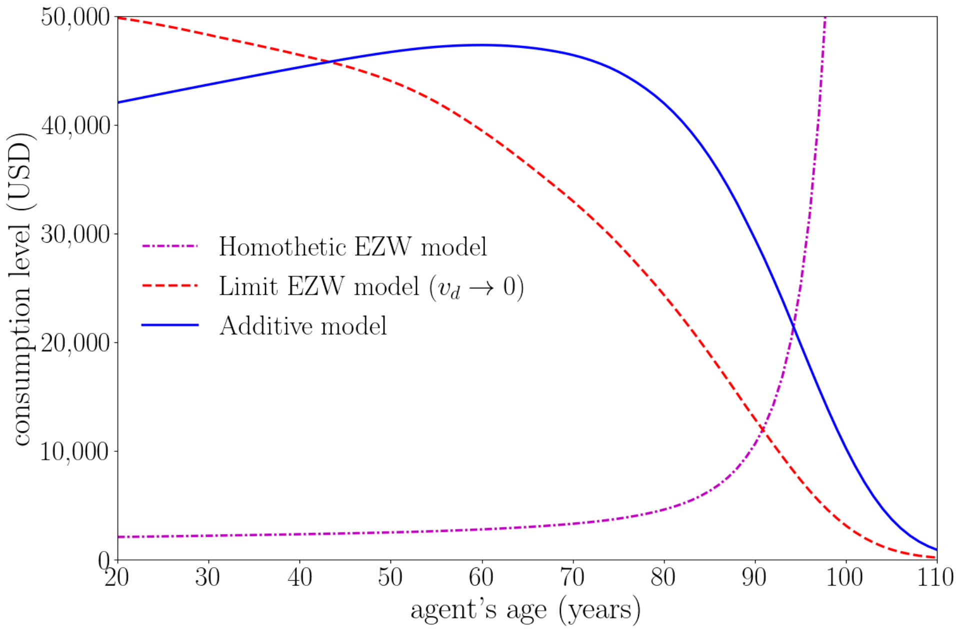


Figure 2: Consumption profiles implied by the additive model, Córdoba and Ripoll's solution (FOC (18)) and the limit EZW model (FOC (19)).

in Figure 2 correspond to those obtained when using the FOC (18), for the so-called "homothetic EZW model", and when using the FOC (19) derived from the (correct) limit of EZW model where $v_d \to 0$. To provide a comparison, we also plot the consumption path implied by the standard additive expected utility model for the same parameter values for $\beta$ and $\sigma$ (the FOC is given by (19) with $\gamma = \sigma$). Lifetime wealth is normalized such that the yearly consumption at age 40 for the additive agent is equal to \$ 45,000. This normalization is of little importance, since preferences are homothetic. The homothetic EZW profile exhibits a consumption level that remains extremely low until age 100, but then sky-rockets after that. This is very different from the consumption paths obtained when using the correct FOC (i.e., the limit model) or the additive specification – both being relatively similar. Note that the y-axis is truncated at \$ 50,000 for greater legibility but under the homothetic EZW model, consumption in fact exceeds \$ 100,000 at age 100 and almost reaches

\$ 15,000,000 at age 110. The predictions of the additive and limit EZW model does not exhibit such extreme behaviors.

Note in addition, that in the limit where $v_d \to 0$, the marginal rate of substitution between survival probabilities and consumption, $\frac{\partial V_t}{\partial \pi_t} / \frac{\partial V_t}{\partial c_t}$ tends towards $\infty$. Thus, even though this limit model – when handled properly – could avoid the counterfactual predictions of CR in terms of consumption smoothing, it would remain inappropriate to discuss value of life matters.[11]

## 2.6 Domain restrictions in Hugonnier et al. (2013)

One way to get around the difficulties mentioned above, in line with what HPSA do in continuous-time, is to assume that survival probabilities remain "large" when age tends towards infinity. Formally, when $\sigma > 1$, $\gamma < 1$, $\pi_t > \beta^{\frac{1-\gamma}{\sigma-1}}$ for all $t \geq 0$ and consumption $c_t$ remains bounded away from zero, the recursion:

$$V_t = \left( (1-\beta) c_t^{1-\sigma} + \beta \pi_t^{\frac{1-\sigma}{1-\gamma}} V_{t+1}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

admits several solutions, one being $V_t = 0$ for all $t$, and the other solutions being given by:

$$V_t = \left[ \beta^{-t} \left( \Pi_{j=0}^{t-1} \pi_j \right)^{\frac{\sigma-1}{1-\gamma}} K + \sum_{s=t}^{\infty} \beta^{s-t} \left( \Pi_{j=t}^{s-1} \pi_j \right)^{\frac{1-\sigma}{1-\gamma}} c_s^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{20}$$

for some constant $K \geq 0$. Mathematically speaking, there is no fundamental problem in using equation (20) for economic analyses. This is basically the route followed by HPSA, in their continuous-time setting. However the economic relevance of imposing $\pi_t > \beta^{\frac{1-\gamma}{\sigma-1}}$ is questionable. Indeed, it requires that life expectancy never goes below $\frac{1}{1-\beta^{\frac{1-\gamma}{\sigma-1}}}$. For example, with $\sigma = 2$, $\gamma = 0.5$ and $\beta = 0.97$, life expectancy should never go below 66 years, whatever the age of the agent. This is of course in contradiction

[11] CR "avoid" the counterfactual predictions of Figure 2 by introducing some age specific health profile whose purpose is to counteract the "cumbersome" variations of the discount factor that arise when using the mistaken limit (12). Besides being ad hoc, this comes at the cost of assuming that health reduces (and not increases) utility.

with what occurs in the real world, where life-expectancy typically drops below 66 years when people reach age 14 according to the HMD 2018 population mortality data. Thus the domain restriction, which is presented as a "technical assumption" in HPSA, helps to avoid degeneracy issues, but is a major shortcoming as it precludes all applications to realistic mortality patterns.[12]

## 2.7 Risk-sensitive preferences

The risk-sensitive specification is the one obtained when $\phi : x \mapsto \frac{1-e^{-kx}}{k}$, providing the recursion shown in (7). Such a specification was initially introduced by Hansen and Sargent (1995) in an infinite-horizon setting and later adapted to the problem of intertemporal choice under uncertain lifespan in two working papers (Bommier, 2014 and Bommier et al., 2020). As shown in Proposition 3, this is, in our setting, the only class of recursive preferences that fulfill OD.[13] Risk-sensitive preferences exhibit preference for early resolution of uncertainty when $k > 0$ and $\beta < 1$ or $k < 0$ and $\beta > 1$ and for late resolution of uncertainty when $k < 0$ and $\beta < 1$ or when $k > 0$ and $\beta > 1$. Indifference for the timing of resolution of uncertainty occurs when $k = 0$ or when $\beta = 1$. In those cases, risk-sensitive preferences actually yield expected utility preferences, the standard additive specification being obtained when $k = 0$ and the multiplicative specification of Bommier (2013) when $k \neq 0$ and $\beta = 1$. As demonstrated in Stanca (2021, Theorem 1), risk-sensitive preferences exhibit positive intertemporal correlation aversion if $k > 0$ and negative intertemporal correlation aversion if $k < 0$, independently of the value of $\beta$. The experimental evidence on intertemporal correlation aversion (see Andersen et al., 2018 and Harrison et al., 2022 for recent evidence using an online experiment in the US during the Covid-19 pandemic) would then support the assumption of a positive $k$.

[12] In HPSA the restriction is formulated in equation (24) of their first theorem.

[13] The "multiplicative preferences" axiomatized in Bommier (2013) can also be viewed as a particular case of risk-sensitive preferences where $\beta$ is set to 1. Such preferences can match empirical consumption profiles and have been used in Bommier and Villeneuve (2012) and Bommier and LeGrand (2014) to study the value of life and the demand for annuities, respectively.

The risk-sensitive specification offers a theoretically appealing framework. OD guarantees that dominated choices are ruled out and offers an intuitive interpretation for the impact of risk aversion. Indeed, risk aversion can then be understood in terms of how bad states are weighted compared to good states when making choice under uncertainty. We illustrate these aspects by deriving results on the discount rate and the value of mortality risk reduction, as well as by providing a numerical example. In the remainder of the section, we assume that the only risk is the mortality risk. As a consequence, the expectation symbol in (7) is no longer needed.

#### 2.7.1 Discount factor

The marginal rate of substitution between consumption in times $t+1$ and $t$ is given by the following expression:

$$\frac{\frac{\partial V_t}{\partial c_{t+1}}}{\frac{\partial V_t}{\partial c_t}} = \frac{c_t^\sigma}{c_{t+1}^\sigma} \beta \frac{\pi_t}{\pi_t + (1-\pi_t) e^{kV_{t+1}}},$$

which therefore implies a discount factor equal to $\beta \frac{\pi_t}{\pi_t+(1-\pi_t)e^{kV_{t+1}}}$. This discount factor simplifies to $\beta\pi_t$ in the standard additive model ($k = 0$) but is otherwise endogenous and has the following characteristics.

**Proposition 4** *The discount factor* $\beta \frac{\pi_t}{\pi_t+(1-\pi_t)e^{kV_{t+1}}}$ *is such that:*

1. *it increases with* $\pi_t$*;*

2. *it decreases with* $k$ *(i.e. higher risk aversion makes people become more impatient) if* $\frac{c_t^{1-\sigma}}{1-\sigma} + u_l > 0$ *for all* $t$*;*

3. *it decreases with continuation utility if* $k > 0$ *(and thus with future consumption, and future survival probabilities if* $\frac{c_t^{1-\sigma}}{1-\sigma} + u_l > 0$ *for all* $t$*);*

The proof can be found in Appendix A.4.

The first point simply states that the higher the mortality risk, the more impatient the agent. This effect is already present in the additive model and extends to the risk-sensitive framework.[14] The second point of Proposition 4 means that more risk averse agents are more impatient, under the condition that life is worth living (in Proposition 5, we show that positive instantaneous utilities imply a positive value of mortality risk reduction). This condition guarantees that in any period the agent sees the possibility of dying at the end of the period as a an adverse outcome, compared to the case where she remains alive. The more risk averse the agent, the less she puts weight on the good state (the one in which she remains alive), which translates into a lower discount rate. Finally, the third point states that when the risk aversion coefficient $k$ is positive and the higher the continuation utility, the more impatient the agent will be. Indeed, when the agent anticipates a bright future, the case where she may die at the end of period $t$ looks much worse in terms of welfare than the case where she may survive. One way to reduce this gap in welfare, and thus reduce risk on lifetime utility, involves consuming more today (and less in the future). The willingness to achieve such a risk reduction is naturally magnified by risk aversion. This translates into a lower discount rate. Note that the last two points are not present in the additive case: with additively separable preferences, there is no effect of $k$ by construction and no effect of continuation utility.

### 2.7.2 Value of mortality risk reduction

The value of mortality risk reduction (MRR) is defined as the marginal rate of substitution between survival and consumption.[15] It quantifies how much consumption in period $t$ the agent is willing to give to diminish her mortality risk from period $t$ to period $t+1$, keeping everything else unchanged. Formally, we have:

$$MRR_t = \frac{\frac{\partial V_t}{\partial \pi_t}}{\frac{\partial V_t}{\partial c_t}}. \tag{21}$$

[14] This contrasts with the messages of CR and HPSA who explain that mortality contributes to impatience (as in our case) when the IES is above one, but reduces impatience when the IES is below one. This difference is directly related to the issues discussed in Sections 2.5 and 2.6.

[15] In the absence of annuity and other mortality-related savings, the value of mortality risk reduction can identically be defined as the marginal rate of substitution between survival and wealth.

Another standard denomination for this marginal rate of substitution is the value of a statistical life (VSL). Due to misunderstandings related to the "Value of Life" denomination, there is a debate about the best terminology to use.[16] Independently of the denomination, this concept is extensively used in cost-benefit analysis for public policy design.

In the risk-sensitive framework, the value of mortality risk reduction (21) has the following expression if $k \neq 0$:

$$MRR_t = \frac{\beta}{k} c_t^{\sigma} \frac{1 - e^{-kV_{t+1}}}{\pi_t e^{-kV_{t+1}} + 1 - \pi_t}. \tag{22}$$

This expression reduces to $MRR_t = \beta z_t^{\sigma} V_{t+1}$ in the additive model ($k = 0$), as is found in papers working with additively separable preferences (Hall et al., 2020 for a recent example). The following proposition summarizes the properties of the value of mortality risk reduction in the risk-sensitive model.

**Proposition 5** *The value of mortality risk reduction of equation (22) is such that:*

1. *it is positive at all dates if* $\frac{c_t^{1-\sigma}}{1-\sigma} + u_l > 0$ *for all* $t$*;*

2. *it increases with continuation utility (and thus with future consumption, and future survival probabilities if* $\frac{c_t^{1-\sigma}}{1-\sigma} + u_l > 0$ *for all* $t$*);*

3. *it increases with* $\pi_t$ *if* $k > 0$*;*

4. *the relationship with* $k$ *is ambiguous in general (even when* $k > 0$ *and* $\frac{c_t^{1-\sigma}}{1-\sigma} + u_l > 0$ *for all* $t$*).*

The proof can be found in Appendix A.5. The first point of Proposition 5 states that having positive instantaneous utilities at all dates is a sufficient condition for having a positive value of mortality risk reduction. The positivity condition on the instantaneous utilities should be interpreted as being alive and consuming $c_t$ is

[16] See https://www.epa.gov/environmental-economics/mortality-risk-valuation for a discussion.

preferred to being dead, and results from our normalization $u_d = 0$.[17] A positive value of mortality risk reduction means that the agent is willing to pay to reduce her mortality probability. A negative value would mean that the agent is willing to pay to increase her mortality probability. Such a feature is systematically obtained with homothetic EZW specifications with an IES below one ($\sigma > 1$) and risk aversion above one ($\gamma > 1$), which explains why they are ruled out in the representation result of Proposition 2.

The second point of the proposition states that the value of mortality risk reduction increases with the continuation utility (in case of survival). The higher the "payoff" in case of survival, the more the agent is willing to pay for enjoying it. This effect is already present in the additive model. This also means that the value of mortality risk reduction increases with future consumption and future survival probabilities (if consumption is high enough to make life better than death).

In order to understand the third and fourth points, which are absent with the additive specification, one has to realize that making investments today to lower tomorrow's mortality is not a plain risk reduction. Of course such investments lower the risk of dying (which is a risk reduction), but they also imply taking the risk of dying tomorrow after having made sacrifices today (which is thus an increase in risk). This latter aspect is more a concern when the risk of dying tomorrow is high (that is when $\pi_t$ is low). We then naturally find that the lower $\pi_t$, the lower the value of mortality risk reduction – which corresponds to the second point of Proposition 5. Also, the fact that making investments to lower mortality involves reducing some risk and generating others is the reason why risk aversion has an ambiguous impact (last point of Proposition 5). This finding and the explanation we provide is fully in line with well-known results in the literature on optimal prevention stating that risk aversion may fail to enhance optimal prevention when the probability of having an accident is not small (see e.g. Dionne and Eeckhoudt, 1985, or Jullien et al., 1999).

[17] Without this normalization, the condition would have been that instantaneous utilities need to be greater than $(1 - \beta)u_d$.

### 2.7.3 A numerical illustration

We complement our theoretical results with simulations showing how risk aversion impacts life-cycle consumption profile and the value of mortality risk reduction when using risk-sensitive preferences. For these simulations, we consider a CRRA utility function of the form $u(c) = \frac{c^{1-\sigma}}{1-\sigma} + u_l$, and set $u_l$ such that the value of mortality risk reduction at age 40 is \$ 10 million in the additive model.[18] The rest of the

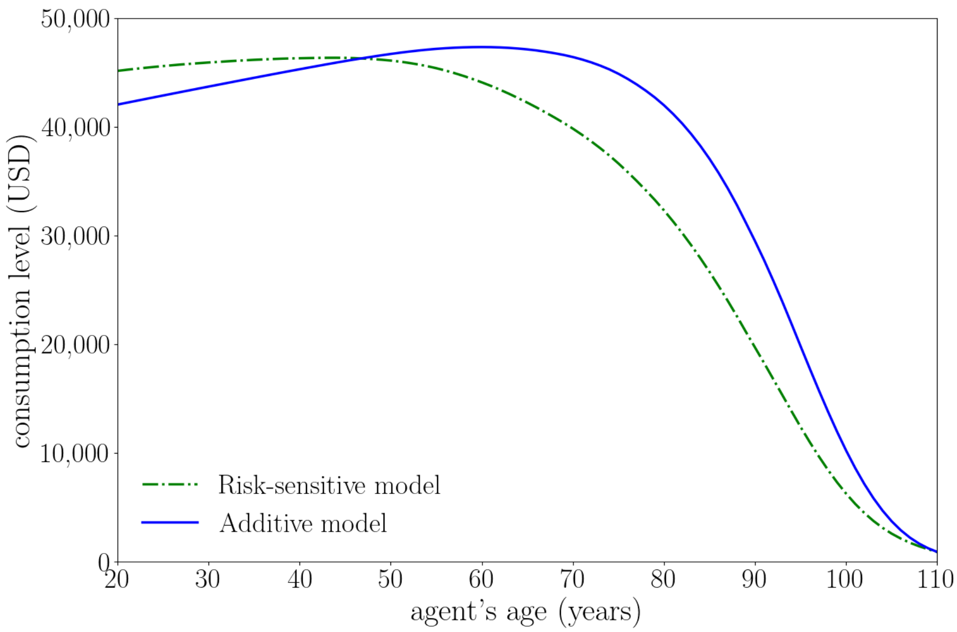


Figure 3: Consumption profiles implied by the additive and risk-sensitive models.

calibration is identical to that in Section 2.5 for the additive model. For risk-sensitive preferences, we use the same calibration as in the additive model and furthermore set $k$ to the value of Bommier et al. (2020) calibrated using annuity data. In order to highlight the role of risk aversion, we contrast the results obtained with a positive $k$ (referred to as the "risk-sensitive model") to those obtained for $k = 0$ (referred to as the "additive model").

Figure 3 reports the consumption paths for both specifications. Since $u_l$ has no impact on consumption choices in the additive model, the additive-model consumption

[18]This is in the range of estimates for the US (in 2021). Viscusi (2021), for example, suggests a value of mortality risk reduction of about \$ 11 million.

profile in Figure 3 is the same as in Figure 2. Both the additive and the risk-sensitive models generate plausible hump-shaped consumption paths. The predictions diverge between the two models, because of the role of risk aversion, which makes the agent more impatient – as stated in the second point of Proposition 4. This is illustrated in Figure 3 by the consumption levels that are larger at younger ages (and lower at older ages) in the risk-sensitive model than in the additive model.

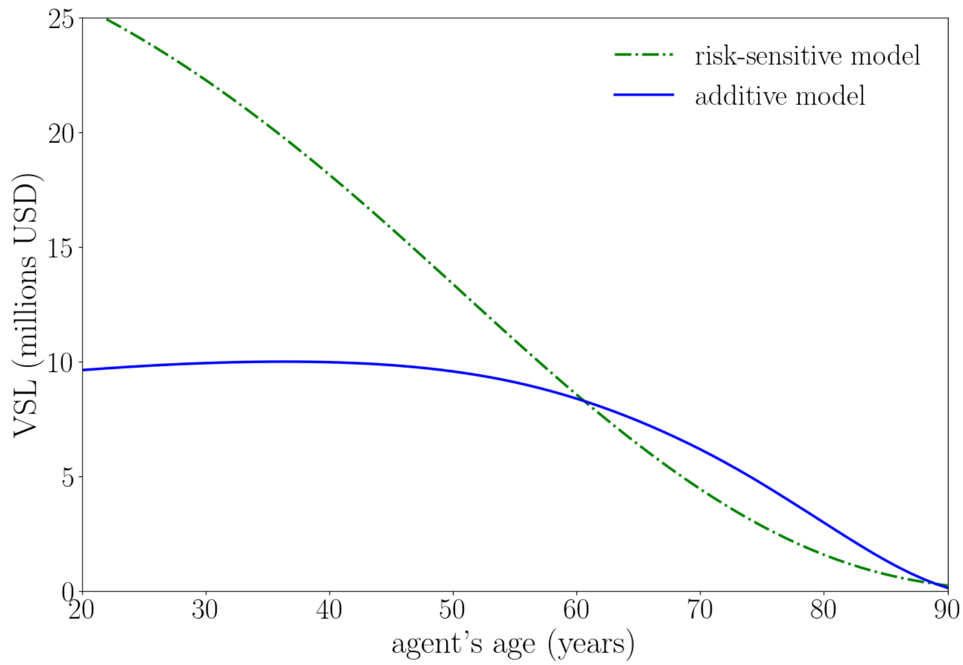


Figure 4: Profiles of value of mortality risk reduction implied by the additive and risk-sensitive models.

We plot in Figure 4 the age-profiles for the value of mortality risk reduction in the additive and risk-sensitive models. We recall that the additive model is calibrated such that the value of mortality risk reduction is $ 10 million at age 40 and that the gap between the two profiles if due to an increase in risk aversion. As stated in Proposition 5, the role of risk aversion is ambiguous. A higher risk aversion contributes to a higher value of mortality risk reduction at younger ages when the mortality risk is low, while it implies a lower value at older ages when the risk is high, in line with the intuition provided in Section 2.7.2.

For both models, the value of mortality reduction tends to decrease with age, reflecting the fact that older people have a shorter life expectancy, thus a smaller stake at play when considering their mortality (the value of mortality risk reduction also depends on the marginal utility of current consumption, which explains why it slightly increases with age at the beginning of the life cycle with the additive model). An important aspect is that the value of mortality risk reduction decreases more rapidly with age with the risk-sensitive specification than with the additive one. The underlying reason is that risk aversion tends to magnify the willingness to avoid particularly dramatic outcomes (such as death at a young age), as compared to the willingness to avoid less adverse outcomes (such as death at an advanced age). We will see that this point turns out to be essential in understanding how risk aversion may alter policy recommendations in the context of epidemic management.

# 3 Application to epidemics

We now illustrate how risk-sensitive preferences provide new insights on the trade-off between consumption and mortality in the context of optimal epidemic mitigation. We build on the framework of Hall et al. (2020) and apply it to the Covid-19 (Section 3.2 ) and 1918 influenza (Section 3.3) pandemics. One reason to focus on these two real-world cases is that they feature highly contrasted age-specific mortality rates. We start by presenting the setup.

## 3.1 General case

We consider a population of size normalized to 1, initially containing agents of different ages $t$ in proportions $(\omega_t)_t$, with $\omega_t \in [0,1]$ and $\sum_t \omega_t = 1$. Agents of age $t$ are endowed with risk-sensitive preferences represented by utility function $V_t$ defined in recursion (7). We assume that a pandemic (either Covid-19 or 1918 influenza in our applications) implies an age-specific impact on survival probabilities, which diminish from $\pi_t$ to $\pi_t - \delta_t$ for one year. A benevolent planner seeks to determine which share $\alpha$ of current consumption agents are willing to give up in exchange for being rid of the excess mortality risk. We further simplify the framework by assuming that consumption, $c$, is constant throughout ages and that agents' discount factor $\beta$ is 1. Let $\lambda = 1 - \alpha$ and denote by $V_t(\lambda, \delta)$ the current utility of an agent of age $t$ whose current consumption is $\lambda c$ (instead of $c$) and her next-period survival $\pi_t - \delta$. One has:

$$V_t(\lambda, \delta) = u(\lambda c) - \frac{1}{k} \log((\pi_t - \delta)e^{-kV_{t+1}(1,0)} + 1 - \pi_t + \delta), \tag{23}$$

where next-period utility is $V_{t+1}(1, 0)$ since the pandemic effects are assumed to last for one year only. The criterion of the benevolent planner is:

$$\begin{aligned} W(\lambda, (\delta_t)_t) &= \sum_t \omega_t V_t(\lambda, \delta_t) \\ &= u(\lambda c) - \frac{1}{k} \sum_t \omega_t \log((\pi_t - \delta_t)e^{-kV_{t+1}(1,0)} + 1 - \pi_t + \delta_t). \end{aligned} \tag{24}$$

The planner seeks to determine how much of the current consumption $c$ can be reduced so as to offset in terms of welfare the extra mortality risk, which corresponds formally to the equivalence $W(1, (\delta_t)_t) = W(\lambda, 0)$, or using (23) and (24) to:

$$u(c) - u(\lambda c) = \frac{1}{k} \sum_t \omega_t \log\left(1 + \delta_t \frac{1 - e^{-kV_{t+1}(1,0)}}{\pi_t e^{-kV_{t+1}(1,0)} + 1 - \pi_t}\right). \tag{25}$$

If we assume that $\delta_t$ and $\lambda$ are both small, we obtain:

$$\alpha = 1 - \lambda \approx \frac{1}{cu'(c)} \sum_t \delta_t \omega_t \frac{1}{k} \frac{1 - e^{-kV_{t+1}(1,0)}}{\pi_t e^{-kV_{t+1}(1,0)} + 1 - \pi_t}, \tag{26}$$

where the latter relationship can be shown to fall back on the expression in Hall et al. (2020, equation (4)) when taking the limit $k \to 0$.

To interpret further equation (26), we can conduct a first-order Taylor expansion in $k$ of expression (25) for $\alpha$. We obtain the following approximation for small $k$:

$$\alpha \approx \underbrace{v \sum_t \delta_t \omega_t \mathbb{E}_{t+1}[\widetilde{T}]}_{\text{additive term}} + v \frac{ku(c)}{2} \left( \sum_t \delta_t \omega_t \left( \underbrace{(2\pi_t - 1)\, \mathbb{E}_{t+1}[\widetilde{T}]^2}_{\text{gain proportional to } \mathbb{E}_{t+1}[\widetilde{T}]^2} - \underbrace{\mathbb{V}_{t+1}[\widetilde{T}]}_{\text{loss due to risk}} \right) \right), \tag{27}$$

where $v = \frac{u(c)}{cu'(c)}$ is, as in Hall et al. (2020), the value of a year of life relative to consumption, $\mathbb{E}_{t+1}[\widetilde{T}]$ the life expectancy at age $t+1$ and $\mathbb{V}_{t+1}[\widetilde{T}]$ the variance of lifespans at age $t+1$. The expression (27) consists of three terms. The first term ("additive term") is the same as in Hall et al. (2020), which is consistent with the fact that the risk-sensitive model reduces to the additive model when $k = 0$. The second term ("gain") is positive when $\pi_t > 0.5$, which is the case for all ages except for very old ages. For instance, in the HMD data we use, it is only at ages greater than 105 that $\pi_t < 0.5$. This term is proportional to the square of life expectancy and reflects that agents with a long life expectancy are willing to pay more to be rid of the additional mortality risk that the epidemic generates (provided their survival probability is high enough). The last term is proportional to the variance of lifespans at age $t+1$: the more uncertain the lifespan, the less the agent is willing to pay to avoid the extra mortality risk. We expect the sum of the two last terms, scaled by the risk aversion parameter $k$, to be positive at younger ages, and negative at older ages. The overall impact of risk aversion is thus not clear-cut and may increase or decrease the value obtained with the additive model, depending on how the epidemic affects younger people as opposed to older people.[19]

## 3.2 The case of Covid-19

We now apply the computations of Section 3.1 to the Covid-19 pandemic. As in Hall et al. (2020), we assume that $u$ is CRRA, with a constant inverse IES set to $\sigma = 2$. We also use their calibration for the consumption level $c$, set to $\$\,45,000$, and for the value of life at age 40, set to $\$\,10.4$m and used to determine $u_l$. The population shares $(\omega_t)_t$ are also those of the US total population in 2018, as reported by the US Census Bureau.[20]

[19] The fact that the effect is positive only at young ages when the survival probability is large enough is in line with our result of Proposition 5 and its illustration in Section 2.7.3. As already mentioned, this is consistent with the literature on optimal prevention (Dionne and Eeckhoudt, 1985, or Jullien et al., 1999).

[20] `https://data.census.gov/cedsci/table?q=population&tid=ACSDP1Y2018.DP05`.

For the Covid-19 age-specific mortality profile $(\delta_t)_t$, Hall et al. (2020) used the data of Ferguson et al. (2020) that was the only reliable source of data available when they wrote their paper. However, these data were estimated very early in the pandemic (March 2020), and we take advantage of more recent estimates that cover a broader set of observations. The data we use are taken from the meta-estimation of Levin et al. (2020), who report an exponential relationship between age and the infection fatality ratio (IFR), which is the proportion of people infected who die from the disease:

$$\log_{10} IFR_t = -3.27 + 0.0524 \times t, \tag{28}$$

where $IFR_t$ is the IFR at age $t$ and $\log_{10}$ is the log in base 10. This implies that the IFR increases by 12.82% every year of age – which is slightly higher than the 11.2% reported in Hall et al. (2020) based on Ferguson et al. (2020) estimates. Regarding the average mortality rate, we use the value of 0.69% which results from the age-specific IFR profile estimated by Levin et al. (2020) applied to the 2018 US population structure and a risk of contracting the Covid-19 of 65%, identical for all ages by assumption. This implies that we have: $\delta_t = 65\% \times IFR_t$. The infection probability of 65% corresponds to a reproductive number $R_0$ of 2.87, which is the mid-point estimate in the meta-review of Billah et al. (2020).

Finally, the survival probabilities $(\pi_t)_t$ are chosen to be those of the US total population in 2018, as reported in the Human Mortality Database.[21] For the risk-sensitive model, we set the value of $k = 0.216$ based on Bommier and LeGrand (2014), who calibrated a risk-sensitive model with $\beta = 1$ to match annuity holdings.

We report in Table 2 the share $\alpha$ of consumption agents are willing to relinquish in order to be rid of the excess mortality risk of Covid-19. We do so for both the risk-sensitive and additive models and for each report the values $\alpha$ obtained from the linear approximation (equation (26)) and from the exact formula (equation (25)).[22]

[21] `https://www.mortality.org/cgi-bin/hmd/country.php?cntr=USA&level=1`. Hall et al. (2020) use mortality data from the Social Security Administration, which differ very slightly from HMD data.

[22] The corresponding expressions for the additive model are simply obtained by taking the limit

When using the additive model, agents are willing to give up 50.4% of their current consumption to avoid Covid-19 mortality risk according to the linear approximation, while this drop in consumption reduces to 33.5% according to the exact formula, which takes into account non-linearities.

When using the risk-sensitive model, the share $\alpha$ of consumption to relinquish is smaller than in the additive setup. With the exact formula, the acceptable drop in consumption equals 28.3% with risk-sensitive preferences, compared to 33.5% for additive ones. Taking risk aversion into account therefore diminishes the share of consumption agents are willing to relinquish, by 5 percentage points approximately (for the exact formula).

| Computational method | Additive model | Risk-Sensitive model |
|---|---|---|
| Linear approximation (26) | 50.4 | 39.6 |
| Exact formula (25) | 33.5 | 28.3 |

Table 2: The share $\alpha$ of consumption (in %) to relinquish to be rid of Covid-19 mortality risk. Computations are based on Levin et al. (2020) data and an average mortality rate of 0.69%

The differences between the results implied by the additive and risk-sensitive models are a direct implication of the role of risk aversion which leads to putting greater weight on more adverse consequences. To illustrate this, we plot in Figure 5 the parameters $(\alpha_t)_t$ of equation (26) as a function of age in both the additive and the risk-sensitive models. Each $\alpha_t$ represents the drop in consumption that a population only made of agents with age $t$ is willing to accept to be rid of an extra mortality risk of 0.1%.[23] It can be seen that the $(\alpha_t)_t$ are decreasing with age for both models, showing that, when assuming a flat consumption profile, younger

$k \to 0$ in (25) and (26).

[23]Note that for a mortality risk higher than 0.1%, the value of $\alpha_t$ for some ages $t$ could be higher than 100%. This simply reflects the limitation of the linear approximation of equation (26) that is only valid for low mortality risks. As can be seen in Table 2, the linear approximation is not very precise in the case of Covid-19 either for the additive or the risk-sensitive models.

agents are more willing to give up consumption than older ones for a given reduction in mortality risk. This reflects the fact that dying young is a more adverse event

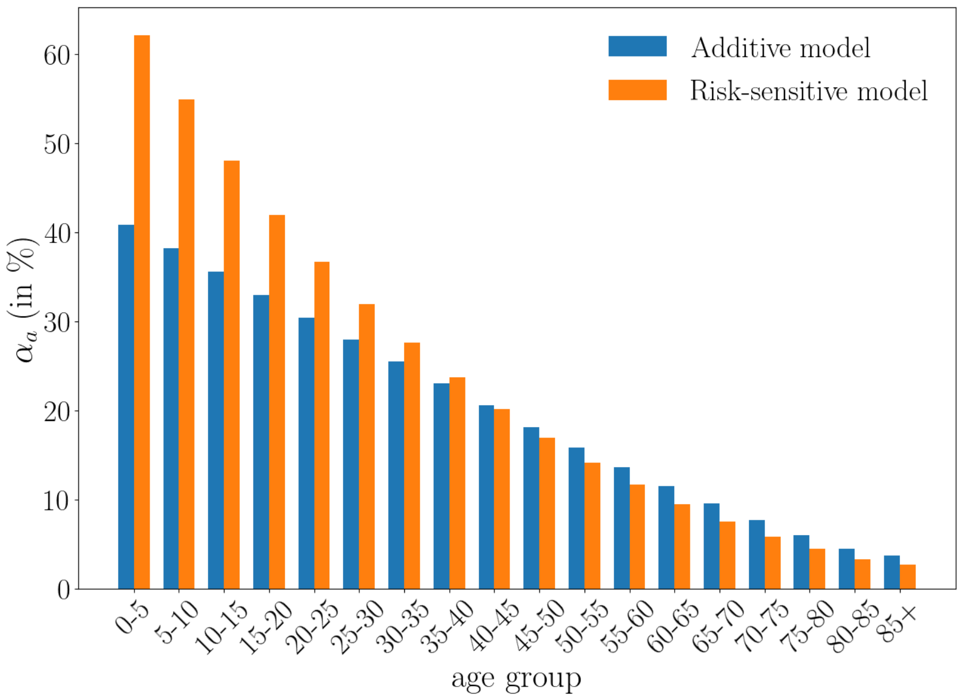


Figure 5: Profiles of the $(\alpha_t)$ parameters as a function of age.

than dying old. However, the difference between additive and risk-sensitive models is that, due to greater risk aversion, short lives are comparatively a greater source of concern in the risk-sensitive model than in the additive model, which makes young agents willing to pay more with risk-sensitive preferences than with additive ones. The opposite holds for older agents. The calibration being performed on the willingness-to-pay for mortality risk reduction at age 40, the differences between models for age groups 35-40 and 40-45 are very modest. Since Covid-19 mainly affects older people, this explains why overall, the acceptable drop in consumption is smaller with the risk-sensitive model than with the additive one.

### 3.3 The case of 1918 influenza

We contrast the results for Covid-19 with those obtained when considering the 1918 influenza, a disease that heavily affected young people. We use the mortality data provided in Taubenberger and Morens (2006) based on Collins (1931). The age-profile of mortality risk is plotted in Figure 6.[24] As can be seen, the age mortality profile has a "W-shape", where young adults are also strongly affected by the disease. This shape is peculiar to the 1918 influenza, since regular influenza epidemics exhibit a U-shape mortality profile.

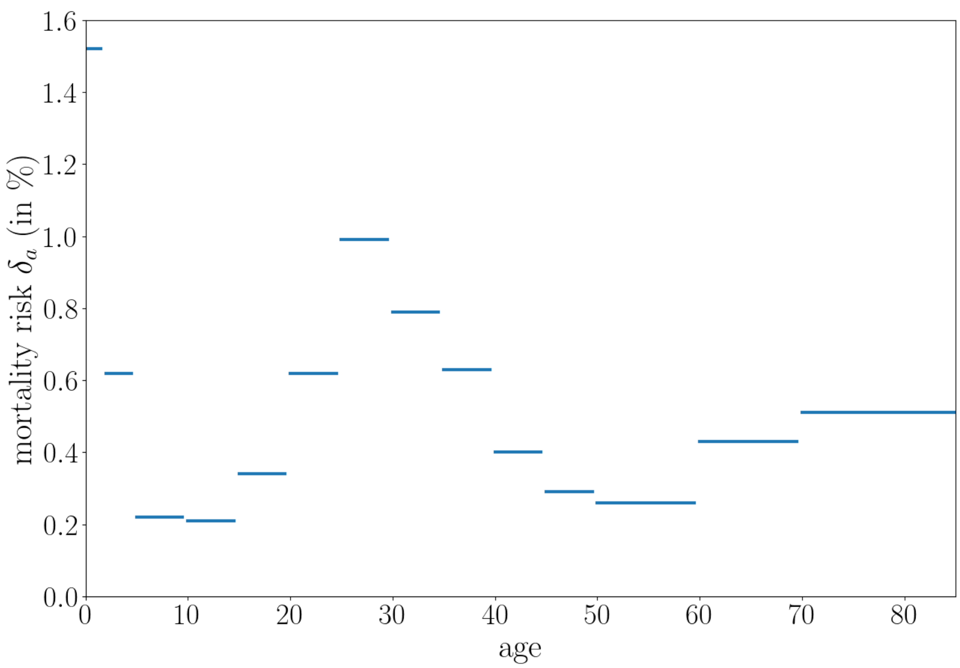


Figure 6: Age-profile of the mortality risk ($\delta_t$) for 1918 influenza.

We report in Table 3 the share of consumption agents are willing to relinquish to be rid of the 1918 influenza mortality risk. This corresponds to the quantity $\alpha$ computed using the exact formula (26). We do not report the values computed with the linear approximation here. Compared to Table 2, we only change the profile ($\delta_t$), keeping the rest of the calibration unchanged. The value in Table 3 should therefore

[24]We directly use here the extra mortality risk implied by 1918 influenza. In the case of Covid-19, such data is not yet available and, as explained in Section 3.2, we estimate it based on the Infection Fatality Ratio (probability of dying once infected) and an infection probability of 65%.

| Mortality pattern | Additive model | Risk-Sensitive model |
|---|---|---|
| 1918 influenza | 56.9 | 60.9 |
| Rescaled 1918 influenza | 64.8 | 68.4 |

Table 3: The share $\alpha$ of consumption (in %) to relinquish to be rid of 1918 influenza mortality risk, computed using the exact formula (26).

be interpreted as the willingness-to-pay to be rid of 1918 influenza mortality risk in the US of 2018 (and not of 1918). To ease the comparison with Covid-19, we consider two mortality patterns: the actual 1918 influenza one, corresponding to the mortality data of Figure 6 and a rescaled version that would yield the same average mortality rate as Covid-19. This "rescaled 1918 influenza scenario" can thus be seen as a fictive pandemic that would feature the same average mortality as Covid-19 but with the age-specific profile of the 1918 influenza.[25]

Unsurprisingly, the values of $\alpha$ for both models are higher for the 1918 influenza than for Covid-19, even when controlling for average mortality. This is due to the fact that younger people suffered comparatively much more from the 1918 influenza that from Covid-19. Furthermore, compared to Table 2, the relative outcomes of the additive and risk-sensitive models are reversed. With Covid-19 affecting older people disproportionately, the additive model tended to overestimate $\alpha$ compared to the risk-sensitive model. With the 1918 influenza also strongly affecting younger people, this is the opposite and the additive model tends to underestimate the value of $\alpha$ compared to the risk-sensitive model.

# 4 Conclusion

Finding the appropriate policy in the case of a health crisis typically amounts to making trade-offs between mortality and consumption (or wealth). The recent Covid-

[25]The average mortality is actually slightly smaller for the 1918 influenza pandemic (about 0.5%) than for the Covid-19 (about 0.69%).

19 crisis has shown how sensitive such issues can be. Risk-reduction measures, such as lock-downs, have been strongly criticized both for being excessive and for not being severe enough. According to the micro-economic tradition, the choice of the social planner should be based on revealed preferences for mortality risk reduction. The difficulty, however, is that estimates on the willingness-to-pay for mortality risk reduction are most often based on samples of workers who, by nature, are not representative of the older and younger populations. Economists are then left with no other option than to make the best extrapolation they can. The standard approach involves using an additive model, which constrains risk aversion to be equal to the inverse of the IES. Such a property is anything but neutral as it drives how particularly negative consequences are weighted compared to not so adverse ones.

The most popular way to disentangle risk aversion and intertemporal substitutability consists of using recursive preferences as initially suggested by Kreps and Porteus (1978) and Epstein and Zin (1989). In the current paper, we have shown how this line of research can contribute to the value-of-life literature. Our message is twofold. First, we highlight that the homothetic EZW specifications are unappealing in the context of mortality risk, constraining in particular the IES to be above one and risk aversion to be below one. The gain in flexibility resulting from the recursive approach entails significant restrictions on key utility parameters. Second, we show that the risk-sensitive preferences initially introduced by Hansen and Sargent (1995), and shown to be the only class of monotone recursive preferences that disentangles risk aversion and IES, can provide new insights for lifecycle analysis. The main benefit of using such preferences is their ability to exhibit greater or lower levels of risk aversion which, unsurprisingly, is something that matters when considering mortality risks. Moreover, the risk-sensitive framework does not constrain the IES (which does not even need to be constant)

In practice, we find that increasing risk aversion leads to exhibiting greater concern for deaths at younger ages, and relatively less for deaths at older ages, reflecting that the death of a young individual is considered as being a more dramatic consequence than the death of an elderly person. We illustrated the relevance of this point by contrasting Covid-19 with the 1918 influenza outbreak, but there are of course many other cases where accounting for risk aversion could yield interesting new insights: one could for example think of the opioid epidemic or of the gun violence public health epidemic.[26] Indirectly, our paper also emphasizes that epidemic management requires the age-distribution of deaths (and not the total number of fatalities) be carefully considered. Any intent to build a welfare relevant unidimensional indicator from this age distribution needs to correctly account for risk aversion.

# Appendix

## A Proofs

We assume that $u$ is defined on $\mathbb{R}_+$ and twice continuously differentiable on $\mathbb{R}_+^* = \mathbb{R}_+ \setminus \{0\}$.

### A.1 Proof of Proposition 1

**Two preliminary lemmas.** First, we prove the following lemma.

**Lemma 1** *Recursive preferences represented with recursion (4) fulfill NSS iff they admit a utility representation $U_t$ where the period utility function $u$ verifies $u(c) > (1-\beta)u_d$ for some $c$.*

**Proof.** If there exists $c$ such that $u(c) > (1-\beta)u_d$ then for all $t < T_{max}$ one has $U_t(c,d) = u(c) + \beta u_d > u_d$ and preferences are NSS. Conversely, assume that $u(c) \leq (1-\beta)u_d$ for all $c$. Then for all elements $(c,m) \in D_{T_{max}-1}$ one has $U_{T_{max}-1}(c,m) =$

[26] See https://www.whitehouse.gov/briefing-room/statements-releases/2021/04/07/fact-sheet-biden-harris-administration-announces-initial-actions-to-address-the-gun-violence-public-health-epidemic/ for a statement from the Biden-Harris administration regarding gun violence in the US (April 7, 2021).

$u(c) + \beta u_d \leq u_d$. This in turn implies, by induction, that for all $t < T_{max}$ and all $(c_t, m) \in D_t$, one has $U(c_t, m) \leq u_d$, in contradiction with NSS. ■

Second we explore the role of monotonicity.

**Lemma 2** *Recursive preferences with recursion (4) are monotone (in the sense stated in Proposition 1) iff they admit a utility representation* $U_t$ *where:*

- *the period utility function* $u$ *is strictly increasing;*
- $u_d$ *and* $\phi(u_d)$ *are finite.*

**Proof.** With our weak order definition, if $c_1 > c_1' \geq 0$, then $(c_1, d) > (c_1', d)$. Monotonicity then implies:

$$U_t(c_1, d) > U_t(c_1', d). \tag{29}$$

By definition, we have $U(c_1', d) = u(c_1') + \beta u_d$. Restricting inequality (29) to $c_1' > 0$, we deduce that since $u$ is continuous, $u(c_1')$ is finite and $u_d < +\infty$. Similarly, using $U_t(c_1, d) = u(c_1) + \beta u_d$, we show that $u_d > -\infty$. We deduce that $u_d$ is finite. For $c_1 > c_1' \geq 0$, inequality (29) is then equivalent to: $u(c_1) > u(c_1')$, which implies that the function $u$ is increasing on $\mathbb{R}_+$. This implies that $u(0)$ is also bounded from above (but can be unbounded from below).

Let us further consider $c_0' > 0$. Monotonicity implies:

$$U_t(c_0', \frac{1}{2}(c_1, d) \oplus \frac{1}{2}d) > U_t(c_0', \frac{1}{2}(c_1', d) \oplus \frac{1}{2}d), \tag{30}$$

or, equivalently:

$$\phi^{-1}\left(\frac{1}{2}\phi(u(c_1) + \beta u_d) + \frac{1}{2}\phi(u_d)\right) > \phi^{-1}\left(\frac{1}{2}\phi(u(c_1') + \beta u_d) + \frac{1}{2}\phi(u_d)\right). \tag{31}$$

Let us assume that $\phi(u_d) = \pm\infty$. In that case, $\phi^{-1}\left(\frac{1}{2}\phi(u(c_1) + \beta u_d) + \frac{1}{2}\phi(u_d)\right)$ and $\phi^{-1}\left(\frac{1}{2}\phi(u(c_1') + \beta u_d) + \frac{1}{2}\phi(u_d)\right)$ are equal and inequality (31) does not hold. We deduce that $\phi(u_d)$ is finite. ■

**Proof of Proposition 1.** We assume that NSS and Monotonicity hold. We have:

$$\begin{cases} U_t(d) = u_d \\ U_t(c_t, m) = u(c_t) + \beta\phi^{-1}(\pi_t(m)E_{m_S}[\phi(U_{t+1})] + (1-\pi_t(m))\phi(u_d)) \end{cases} \tag{32}$$

Since from Lemma 2, $u_d$ is finite, we can set for all $t$ and for all $m \in D_t$, $\tilde{U}_t(m) = U_t(m) - u_d$, as well as $\tilde{\phi}(x) = \phi(x + u_d)$, which means that $\tilde{\phi}^{-1}(y) = \phi^{-1}(y) - u_d$. Recursion (32) then becomes:

$$\begin{cases} \tilde{U}_t(d) = 0, \\ \tilde{U}_t(c_t, m) = u(c_t) + \beta\tilde{\phi}^{-1}(\pi_t(m)E_{m_S}[\tilde{\phi}(\tilde{U}_{t+1})] + (1-\pi_t(m))\tilde{\phi}(0)), \end{cases}$$

Since $\tilde{U}_t$ represents the same preferences as $U_t$, we can set $u_d = 0$ without loss of generality.

Furthermore, since $\phi(u_d)$ is finite (Lemma 2), so is $\tilde{\phi}(0)$. We can define: $\hat{\phi}(x) = \tilde{\phi}(x) - \tilde{\phi}(0)$ – which means that $\hat{\phi}^{-1}(y) = \tilde{\phi}^{-1}(x + \tilde{\phi}(0))$. Thus, recursion (32) finally becomes:

$$\begin{cases} \tilde{U}_t(d) = 0, \\ \tilde{U}_t(c_t, m) = u(c_t) + \beta\hat{\phi}^{-1}(\pi_t(m)E_{m_S}[\hat{\phi}(\tilde{U}_{t+1})]), \end{cases}$$

Dropping the tilde and hat decoration, we will obtain recursion (6). Furthermore, observe that $\phi$ is defined up to a positive multiplicative factor: if preferences are represented by $\phi$, they will also be represented by $\mu\phi$ for any $\mu > 0$.

## A.2 Proof of Proposition 3

We consider $c_0, c_1 > 0$ and $\pi \in (0, 1)$. Monotonicity implies that $u$ is strictly increasing and that $u^{-1}$ exists and is unique. For any $c > 0$, we define the function $\eta_{c_0,c_1}$ as follows:

$$\eta_{c_0,c_1}(c) = u^{-1}\left(\phi^{-1}\left(\pi^{-1}\phi\left(\beta^{-1}(u(c_0) - u(c)) + \phi^{-1}\left(\pi\phi(u(c_1))\right)\right)\right)\right), \tag{33}$$

We can observe that by definition:

$$\eta_{c_0,c_1}(c_0) = u^{-1}\left(u(c_1)\right) = c_1, \tag{34}$$

and

$$U_t(c, \pi(\eta_{c_0,c_1}(c), d) \oplus (1-\pi)d) = U_t(c_0, \pi(c_1, d) \oplus (1-\pi)d). \tag{35}$$

The former relationship can equivalently be written as:

$$u(c_0) + \beta\phi^{-1}\left(\pi\phi(u(c_1))\right) = u(c) + \beta\phi^{-1}\left(\pi\phi(u(\eta_{c_0,c_1}(c)))\right). \tag{36}$$

From (33) and (34), since $u$ and $\phi$ are continuously differentiable, there exists a neighborhood $\widetilde{B}_{c_0}$ of $c_0$, where $\eta_{c_0,c_1}$ exists and is continuously differentiable.

Let $c_1' > 0$ and $c_1' \neq c_1$. Similarly, we can find another neighborhood $\widehat{B}_{c_0}$ of $c_0$ such that the function $\eta_{c_0,c_1'}(c)$ defined as in (33) is continuously differentiable on $\widehat{B}_{c_0}$ and verifies for $c \in \widehat{B}_{c_0}$:

$$u(c_0) + \beta\phi^{-1}\left(\pi\phi(u(c_1'))\right) = u(c) + \beta\phi^{-1}\left(\pi\phi(u(\eta_{c_0,c_1'}(c)))\right). \tag{37}$$

We define $B_{c_0} = \widehat{B}_{c_0} \cap \widetilde{B}_{c_0}$, which is a non-empty open set. OD implies that equalities (36) and (37) lead to, for all $c \in B_{c_0}$:

$$\begin{aligned} u(c_0) + \beta\phi^{-1}\left(\frac{\pi}{2}\left(\phi(u(c_1)) + \phi(u(c_1'))\right)\right) = \\ u(c) + \beta\phi^{-1}\left(\frac{\pi}{2}\left(\phi(u(\eta_{c_0,c_1}(c))) + \phi(u(\eta_{c_0,c_1'}(c)))\right)\right). \end{aligned} \tag{38}$$

We compute the derivatives of equation (36) and its counterpart for $\eta_{c_0,c_1'}$, as well as of (38). We obtain for $c \in B_0$:

$$\begin{aligned} 0 &= u'(c) + \beta\pi\frac{\partial\eta_{c_0,c_1}(c)}{\partial c}u'(\eta_{c_0,c_1}(c))\phi'(u(\eta_{c_0,c_1}(c)))(\phi^{-1})'\left(\pi\phi(u(\eta_{c_0,c_1}(c)))\right), \\ 0 &= u'(c) + \beta\pi\frac{\partial\eta_{c_0,c_1'}(c)}{\partial c}u'(\eta_{c_0,c_1'}(c))\phi'(u(\eta_{c_0,c_1'}(c)))(\phi^{-1})'\left(\pi\phi(u(\eta_{c_0,c_1'}(c)))\right), \\ 0 &= u'(c) + \frac{1}{2}(\phi^{-1})'\left(\frac{\pi}{2}\left(\phi(u(\eta_{c_0,c_1}(c))) + \phi(u(\eta_{c_0,c_1'}(c)))\right)\right) \\ &\times\left(\beta\pi\frac{\partial\eta_{c_0,c_1}(c)}{\partial c}u'(\eta_{c_0,c_1}(c))\phi'(u(\eta_{c_0,c_1}(c))) + \beta\pi\frac{\partial\eta_{c_0,c_1'}(c)}{\partial c}u'(\eta_{c_0,c_1'}(c))\phi'(u(\eta_{c_0,c_1'}(c)))\right). \end{aligned}$$

After substitution, we obtain since $u'(c) > 0$ and using $(\phi^{-1})'$;

$$\frac{1}{(\phi^{-1})'\left(\frac{\pi}{2}\left(\phi(u(\eta_{c_0,c_1}(c))) + \phi(u(\eta_{c_0,c_1'}(c)))\right)\right)} =$$
$$\frac{1}{2}\frac{1}{(\phi^{-1})'\left(\pi\phi(u(\eta_{c_0,c_1}(c)))\right)} + \frac{1}{2}\frac{1}{(\phi^{-1})'\left(\pi\phi(u(\eta_{c_0,c_1'}(c)))\right)}.$$

We deduce that $\frac{1}{(\phi^{-1})'}$ is affine locally (see Aczél, 1966). Note that by changing $\pi$ and $c_0$ we can cover the whole definition space of $\frac{1}{(\phi^{-1})'}$ and by continuity guarantee that $\frac{1}{(\phi^{-1})'}$ is actually affine globally. We deduce that there exist $k$ and $y_0$, such that $\frac{1}{(\phi^{-1})'(y)} = -k(y - y_0)$. After integration, we find that there exists $\phi_0$, such that: $\phi^{-1}(y) = -\frac{1}{k}\ln(\frac{y-y_0}{\phi_0})$, which implies $\phi(x) = \phi_0 e^{-kx} + y_0$. We further normalize $\phi$ using $\phi(0) = 0$ and $\phi'(0) = 1$ and obtain:

$$\phi(x) = \frac{1}{k}\left(1 - e^{-kx}\right).$$

## A.3 Proof of Proposition 2

### A.3.1 First step.

We consider $c_0, c_1 > 0$. We have: $U_t(c_0, (c_1, d)) = u(c_0) + \beta u(c_1)$. The marginal rate of substitution between $c_0$ and $c_1$, denoted by $MRS_{c_0,c_1}$ can be written as follows:

$$MRS_{c_0,c_1} = \beta\frac{u'(c_1)}{u'(c_0)}. \tag{39}$$

Homotheticity implies that $MRS_{c_0,c_1} = MRS_{\lambda c_0,\lambda c_1}$.[27] From (39), $\frac{u'(\lambda c_1)}{u'(\lambda c_0)}$ is independent of $\lambda$. Computing the log-derivative in $\lambda$ for $\lambda = 1$ yields: $c_1\frac{u''(c_1)}{u'(c_1)} = c_0\frac{u''(c_0)}{u'(c_0)}$, which means that $c\frac{u''(c)}{u'(c)}$ is constant and $u$ is CRRA. There exists $K > 0$, $\sigma$, and $u_l$, such that:

$$u(c) = \begin{cases} K\frac{c^{1-\sigma}}{1-\sigma} + u_l & \text{if } \sigma \neq 1, \\ \ln(c) + u_l & \text{otherwise.} \end{cases} \tag{40}$$

[27] Let $c_0'$ and $c_1'$ such that $u(c_0) + \beta u(c_1) = u(c_0') + \beta u(c_1')$. We obtain $u'(c_0)\frac{\partial c_0}{\partial c_1} + \beta u'(c_1) = 0$. Using homotheticity, we obtain that for any $\lambda$, $u(\lambda c_0) + \beta u(\lambda c_1) = u(\lambda c_0') + \beta u(\lambda c_1')$, which yields $u'(\lambda c_0)\frac{\partial(\lambda c_0)}{\partial(\lambda c_1)} + \beta u'(\lambda c_1) = 0 = u'(\lambda c_0)\frac{\partial c_0}{\partial c_1} + \beta u'(\lambda c_1)$.

### A.3.2 Second step.

We consider $c_0, c_1 > 0$ and $\pi \in (0,1)$. We have:

$$U_t(c_0, \pi(c_1,d) \oplus (1-\pi)(c_2,d)) = u(c_0) + \beta\phi^{-1}\left(\pi\phi(u(c_1)) + (1-\pi)\phi(u(c_2))\right).$$

The marginal rate of substitution between $c_0$ and $c_1$– that we still denote by $MRS_{c_0,c_1}$ – is then as follows:

$$MRS_{c_0,c_1} = \beta\pi\frac{u'(c_1)}{u'(c_0)}\frac{\phi'(u(c_1))}{\phi'\left(\phi^{-1}\left(\pi\phi(u(c_1)) + (1-\pi)\phi(u(c_2))\right)\right)}.$$

Homotheticity implies $MRS_{c_0,c_1} = MRS_{\lambda c_0,\lambda c_1}$. Since we already proved that $\frac{u'(c_1)}{u'(c_0)} = \frac{u'(\lambda c_1)}{u'(\lambda c_0)}$, this implies that $\frac{\phi'(u(\lambda c_1))}{\phi'(\phi^{-1}(\pi\phi(u(\lambda c_1))+(1-\pi)\pi\phi(u(c_2))))}$ is independent of $\lambda$. Similarly, considering the MRS between $c_0$ and $c_2$, we obtain that $\frac{\phi'(u(\lambda c_2))}{\phi'(\phi^{-1}(\pi\phi(u(\lambda c_1))+(1-\pi)\pi\phi(u(c_2))))}$ is independent of $\lambda$. Taking the ratio, we deduce that $\frac{\phi'(u(\lambda c_1))}{\phi'(u(\lambda c_2))}$ is independent of $\lambda$. Taking the log derivative yields:

$$c_1 u'(\lambda c_1)\frac{\phi''(u(\lambda c_1))}{\phi'(u(\lambda c_1))} = c_2 u'(\lambda c_2)\frac{\phi''(u(\lambda c_2))}{\phi'(u(\lambda c_2))}, \tag{41}$$

or equivalently that $cu'(c)\frac{\phi''(u(c))}{\phi'(u(c))}$ is constant. There are two cases.

1. There exists $c$, such that $\phi''(u(c)) = 0$. In that case, from (41), we deduce that $\phi''$ is null everywhere. Since $\phi(0) = 0$ and we impose $\phi'(0) = 1$, we obtain $\phi(x) = x$. From (40), we deduce that (6) becomes:

$$U_t(c_t, m) = \begin{cases} K\frac{c^{1-\sigma}}{1-\sigma} + u_l + \beta\pi_t U_{t+1}, \\ K\ln(c) + u_l + \beta\pi_t U_{t+1}, \end{cases}$$

   which is the standard Yaari (1965) model with a CRRA utility function.

2. For all $c$, $\phi''(u(c)) \neq 0$. There are two cases depending on the functional form of $u$ in (40).

   (a) $u(c) = K\frac{c^{1-\sigma}}{1-\sigma} + u_l$ for some $K > 0$, $\sigma \neq 1$, and $u_0$. Using (41), we obtain that there exists $\rho$, such that for all $c$:

$$K\frac{c^{1-\sigma}}{1-\sigma}\frac{\phi''(K\frac{c^{1-\sigma}}{1-\sigma}+u_l)}{\phi'(K\frac{c^{1-\sigma}}{1-\sigma}+u_l)}=\rho-1$$

or with $y=K\frac{c^{1-\sigma}}{1-\sigma}+u_l$: $(y-u_l)\frac{\phi''(y)}{\phi'(y)}=\rho-1$, which yields with $\phi(0)=0$:

$$\phi(y)=\frac{\phi_0\left((y-u_l)^\rho-(-u_l)^\rho\right)}{\kappa}. \tag{42}$$

We now consider

$$U_t(c_0,\pi(c_1,d)\oplus(1-\pi)d)=u(c_0)+\beta\phi^{-1}\left(\pi\phi(u(c_1))\right)$$

and the MRS between $c_0$ and $c_1$. Using similar steps as above and setting $u_1=\phi^{-1}\left(\pi\phi(u(\lambda c_1))\right)$:

$$\phi(u(\lambda c_1))\frac{\phi''(u(\lambda c_1))}{\phi'(u(\lambda c_1))^2}=\phi(u_1)\frac{\phi''(u_1)}{\phi'(u_1)^2}.$$

This implies that there exist a neighborhood $B$ and a constant $C$, such that for all $y\in B$, we have:

$$\frac{\phi(y)\phi''(y)}{\phi'(y)^2}=C. \tag{43}$$

Using (42), we deduce for all $y\in B$, $1-\frac{(-u_l)^\rho}{(y-u_l)^\rho}=\frac{C\kappa}{\rho-1}$, which imposes $u_l=0$. Using (42) and $\phi'(1)=\rho$ for the scaling normalization, we obtain $\phi(y)=y^\rho$, with $\rho>0$, since $\phi$ is increasing. Recursion (6) becomes:

$$U_t(c_t,m)=K\frac{c_t^{1-\sigma}}{1-\sigma}+\beta(\pi_t E[U_{t+1}^\rho])^{\frac{1}{\rho}},$$

where we can set $K=1$ without loss of generality since the preferences represented by $U_t$ and $K^{-1}U_t$ are the same. We finally obtain:

$$U_t(c_t,m)=\frac{c_t^{1-\sigma}}{1-\sigma}+\beta(\pi_t E[U_{t+1}^\rho])^{\frac{1}{\rho}},\ \text{with } \rho>0.$$

(b) $u(c)=K\ln(c)+u_l$ for some $K>0$ and $u_l$. We use similar steps as in the previous case. Equation (41) implies that there exists $k$, such that $\frac{\phi''(y)}{\phi'(y)} = -k$ for all $y$, or after integration with $\phi(0) = 0$ and $\phi'(0) = 1$ (that we can impose without loss of generality): $\phi(y) = \frac{1-e^{-ky}}{k}$. Furthermore, equality (43) still holds and yields: $1 - e^{ky} = C$ for all $y$, which imposes $k = 0$. We fall back on the additive case.

## A.4 Proof of Proposition 4

As a preliminary, we prove the following Lemma.

**Lemma 3** *If $u(c_t) > 0$ for all $t \in \{0, \ldots, T_{max} - 1\}$, we have $V_t > 0$ for all $t \in \{0, \ldots, T_{max} - 1\}$.*

The proof goes by backward induction. For $t = T_{max} - 1$, we have $V_{T_{max}-1} = u(c_{T_{max}-1}) > 0$ by assumption. Let $t \in \{0, \ldots, T_{max} - 2\}$ and assume that the result holds for $t + 1$. We have by recursion:

$$V_t = u(c_t) - \frac{\beta}{k} \log(\pi_t e^{-kV_{t+1}} + 1 - \pi_t).$$

Since $x \mapsto -\frac{\beta}{k} \log(\pi_t e^{-kx} + 1 - \pi_t)$ is increasing (its derivative is $\beta \frac{\pi_t e^{-kx}}{\pi_t e^{-kx} + 1 - \pi_t} \geq 0$) and $V_{t+1} \geq 0$ by induction hypothesis, we deduce that $V_t \geq u(c_t) > 0$. This concludes the induction and the proof of Lemma 3.

The proof of the first point of Proposition 4 is straightforward as $V_{t+1}$ is independent of $\pi_t$. So, $\frac{\partial \Delta_t}{\partial \pi_t} = \beta \frac{e^{kV_{t+1}}}{(\pi_t + (1-\pi_t)e^{kV_{t+1}})^2} > 0$.

We now turn to the proof of the second point. From the expression of the discount factor $\Delta_t = \beta \frac{\pi_t}{\pi_t + (1-\pi_t)e^{kV_{t+1}}}$, we deduce that the sign of $\frac{\partial \Delta_t}{\partial k}$ is the same as the one of $\frac{\partial(-kV_{t+1})}{\partial k}$.[28] Using the recursion (7) and defining $W_t = -kV_t$, we deduce:

$$W_{T_{max}-1} = -ku(c_{T_{max}-1}), \quad (44)$$

$$W_t = -ku(c_t) + \beta \log(\pi_t e^{W_{t+1}} + 1 - \pi_t) \text{ for } t < T_{max} - 1. \quad (45)$$

[28]Though uncertain, the horizon in our setup is finite. So, the differentiability properties of intertemporal utility function (such as $V_t$) directly comes from the differentiability properties of the period utility function.

We show by backward induction that $\frac{\partial W_t}{\partial k} < 0$ for all $t$. For $t = T_{max} - 1$, we deduce from (44):

$$\frac{\partial W_{T_{max}-1}}{\partial k} = -u(c_{T_{max}-1}) < 0,$$

because we assume $u(c_t) > 0$ for all $t$. Let $t \in \{0, \ldots, T_{max} - 2\}$ and assume that $\frac{\partial W_{t+1}}{\partial k} < 0$. Using (45), we get:

$$\frac{\partial W_t}{\partial k} = -u(c_t) + \frac{\beta \pi_t \frac{\partial W_{t+1}}{\partial k}}{\pi_t e^{W_{t+1}} + 1 - \pi_t},$$

which implies $\frac{\partial W_t}{\partial k} < 0$ because of the induction hypothesis and the assumption that $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + u_l > 0$ for all $t$. This concludes the induction and we have $\frac{\partial W_t}{\partial k} < 0$ for all $t$.

For the third point, we directly have $\frac{\partial \Delta_t}{\partial V_{t+1}} = \frac{-\beta k(1-\pi_t)e^{kV_{t+1}}}{(\pi_t + (1-\pi_t)e^{kV_{t+1}})^2} < 0$ if $k > 0$. Let us prove that $V_{t+1}$ is increasing with future consumption and future survival probabilities. Let $t \in \{0, \ldots, T_{max} - 1\}$ and $1 \leq s \leq T_{max} - t - 1$. We first show that $\frac{\partial V_{t+1}}{\partial c_{t+s}} > 0$. Start with observing that because the utility defined in (7) is independent of the past, we have $\frac{\partial V_{t+s+1}}{\partial c_{t+s}} = 0$. We now prove that $\frac{\partial V_{t+s'}}{\partial c_{t+s}} > 0$ for all $s' \in \{1, \ldots, s\}$ by backward induction on $s'$. For $s' = s$, we have using $\frac{\partial V_{t+s+1}}{\partial c_{t+s}} = 0$:

$$\frac{\partial V_{t+s}}{\partial c_{t+s}} = u'(c_{t+s}) > 0.$$

Let $s' \in \{1, \ldots, s-1\}$ and assume that $\frac{\partial V_{t+s'+1}}{\partial c_{t+s}} > 0$. We have using (7):

$$\frac{\partial V_{t+s'}}{\partial c_{t+s}} = \frac{\beta e^{-kV_{t+s'+1}}}{\pi_{t+s} e^{-kV_{t+s'+1}} + 1 - \pi_{t+s}} \frac{\partial V_{t+s'+1}}{\partial c_{t+s}} > 0, \tag{46}$$

as a consequence of the induction hypothesis and $u(c_t) > 0$ for all $t$. This concludes the induction and we have $\frac{\partial V_{t+s'}}{\partial c_{t+s}} > 0$ for all $s' \in \{1, \ldots, s\}$. In particular, for $s' = 1$, $\frac{\partial V_{t+1}}{\partial c_{t+s}} > 0$ where $t \in \{0, \ldots, T_{max} - 1\}$ and $1 \leq s \leq T_{max} - t - 1$ are arbitrary.

The proof is similar for probabilities. We fix $t \in \{0, \ldots, T_{max} - 1\}$ and $1 \leq s \leq T_{max} - t - 1$. Since we have $\frac{\partial V_{t+s+1}}{\partial \pi_{t+s}} = 0$, we will prove that $\frac{\partial V_{t+s'}}{\partial \pi_{t+s}} > 0$ for all

$s' \in \{1, \ldots, s\}$ by backward induction on $s'$. For $s' = s$:

$$\frac{\partial V_{t+s}}{\partial \pi_{t+s}} = \frac{\beta}{k} \frac{1 - e^{-kV_{t+s+1}}}{\pi_t e^{-kV_{t+s+1}} + 1 - \pi_t} > 0.$$

Let $s' \in \{1, \ldots, s-1\}$ and assume that $\frac{\partial V_{t+s'+1}}{\partial c_{t+s}} > 0$. We have: $\frac{\partial V_{t+s'}}{\partial \pi_{t+s}} = \frac{\beta e^{-kV_{t+s'+1}}}{\pi_{t+s} e^{-kV_{t+s'+1}} + 1 - \pi_{t+S}} \frac{\partial V_{t+s'+1}}{\partial \pi_{t+s}} > 0$ by induction hypothesis. This concludes the induction.

## A.5 Proof of Proposition 5

The first point is implied by Lemma 3 and the fact that $V \mapsto \frac{\beta}{k} c_t^{\sigma} \frac{1 - e^{-kV}}{\pi_t e^{-kV} + 1 - \pi_t}$ is increasing (independently of the sign of $k$).

For the second point, we have from (22):

$$\frac{\partial MRR_t}{\partial V_{t+1}} = \beta z_t^{\sigma} \frac{e^{-kV_{t+1}}}{(1 - \pi_t(1 - e^{-kV_{t+1}}))^2} > 0,$$

since $k > 0$ and $u(c_t) > 0$ for all $t$. We conclude the third point on future consumption and probabilities using the same arguments as in the proof of Proposition 4 in Appendix A.4.

For the third point, using the expression of $MRR_t$ in equation (22), we have:

$$\frac{\partial MRR_t}{\partial \pi_t} = \frac{\beta}{k} c_t^{\sigma} \frac{(1 - e^{-kV_{t+1}})^2}{(1 - \pi_t(1 - e^{-kV_{t+1}}))^2},$$

whose sign is the sign of $k$. This proves the second point.

We now turn to the last point. Let assume $k > 0$ and $MRR_t > 0$. From (22), we have:

$$\frac{1}{MRR_t} \frac{\partial MRR_t}{\partial k} = -\frac{1}{k} + \frac{e^{-kV_{t+1}}}{(1 - \pi_t(1 - e^{-kV_{t+1}}))(1 - e^{-kV_{t+1}})} \frac{\partial (kV_{t+1})}{\partial k}.$$

The first term $(-1/k)$ is negative while the second term is positive (from $\frac{\partial W_t}{\partial k} < 0$ as proved in Section A.4 and where $W_t$ is defined in equations (44)–(44)). The overall sign is ambiguous.

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