Online Appendix for: Uncovering Asset Market Participation from Household Consumption and Income

July 18, 2023

1 Proof of Proposition 1

For bond holdings, we compute the marginal rate of substitution $\mathbb{E}_t[\left(c^i_{x,t+1}/c^i_{x,t}\right)^{-\gamma_x}|x]$ at date t for a household of type x (conditional on its type). We obtain:

$$
\begin{aligned}
&\mathbb{E}_t[\exp\{-\gamma_x \Delta \log c^i_{x,t+1}\}|x] = \\
&\quad = \frac{1}{\beta_x R^f_{t+1}} \mathbb{E}_t\left[\exp\left\{-\kappa_x \omega_{t+1} - \sigma_x \varepsilon^i_{t+1} - \psi_x \sqrt{\omega_{t+1}} u_{t+1}\right\}\right] \\
&\quad = \frac{1}{\beta_x R^f_{t+1}} \mathbb{E}_t\left[\exp\left\{\sigma_x^2/2 - \kappa_x \omega_{t+1} - \psi_x \sqrt{\omega_{t+1}} u_{t+1}\right\}\right] \\
&\quad = \frac{1}{\beta_x R^f_{t+1}} \mathbb{E}_t\left[\exp\left\{\sigma_x^2/2 + (\psi_x^2/2 - \kappa_x)\omega_{t+1}\right\}\right].
\end{aligned}
$$

2 Proof of Proposition 2

For stock holdings, we compute the quantity $\mathbb{E}_t[R^s_{t+1}\left(c^i_{x,t+1}/c^i_{x,t}\right)^{-\gamma_x}|x]$

$$
\begin{aligned}
&\mathbb{E}_t[\exp\{\log R^s_{t+1} - \gamma_x \Delta \log c^i_{x,t+1}\}|x] = \\
&\quad = \frac{1}{\beta_x} \mathbb{E}_t\left[\exp\left\{\mu\,\omega_{t+1} + \sqrt{\omega_{t+1}} u_{t+1} - \kappa_x \omega_{t+1} - \sigma_x \varepsilon^i_{t+1} - \psi_x \sqrt{\omega_{t+1}} u_{t+1}\right\}\right] \\
&\quad = \frac{1}{\beta_x} \mathbb{E}_t\left[\exp\left\{\sigma_x^2/2 + \mu\,\omega_{t+1} - \kappa_x \omega_{t+1} + (1-\psi_x)\sqrt{\omega_{t+1}} u_{t+1}\right\}\right] \\
&\quad = \frac{1}{\beta_x} \mathbb{E}_t\left[\exp\left\{\sigma_x^2/2 + (\mu - \kappa_x + 1/2 + \psi_x^2/2 - \psi_x)\omega_{t+1}\right\}\right] \\
&\quad = \frac{1}{\beta_x} \mathbb{E}_t\left[\exp\left\{\sigma_x^2/2 + (\mu + \frac{1}{2} - \psi_x + \frac{\psi_x^2}{2} - \kappa_x)\omega_{t+1}\right\}\right].
\end{aligned}
$$

3 Monte Carlo results

We assess the finite sample accuracy of our estimation method by Monte Carlo simulation. The structural parameters are the II empirical parameter estimates reported in the main paper. These II estimates are then plugged into the limited participation model to generate 100 samples, of the same size as our empirical data. Finally, for each of the 100 data samples, we compute the II estimates of the 24 parameters.

Our results are summarized in the boxplots of Figures 3–3.[1] True parameter values

[1] Boxes provide the first, second, and third quartiles of the II estimates and whiskers provide the farthest

Figure 1: **Boxplots of 100 MC estimates of the structural parameters β_n, β_b, β_s, γ_n, γ_b, γ_s, κ_n, κ_b, κ_s, σ_n, σ_b, and σ_s. True parameters are reported with horizontal lines.**

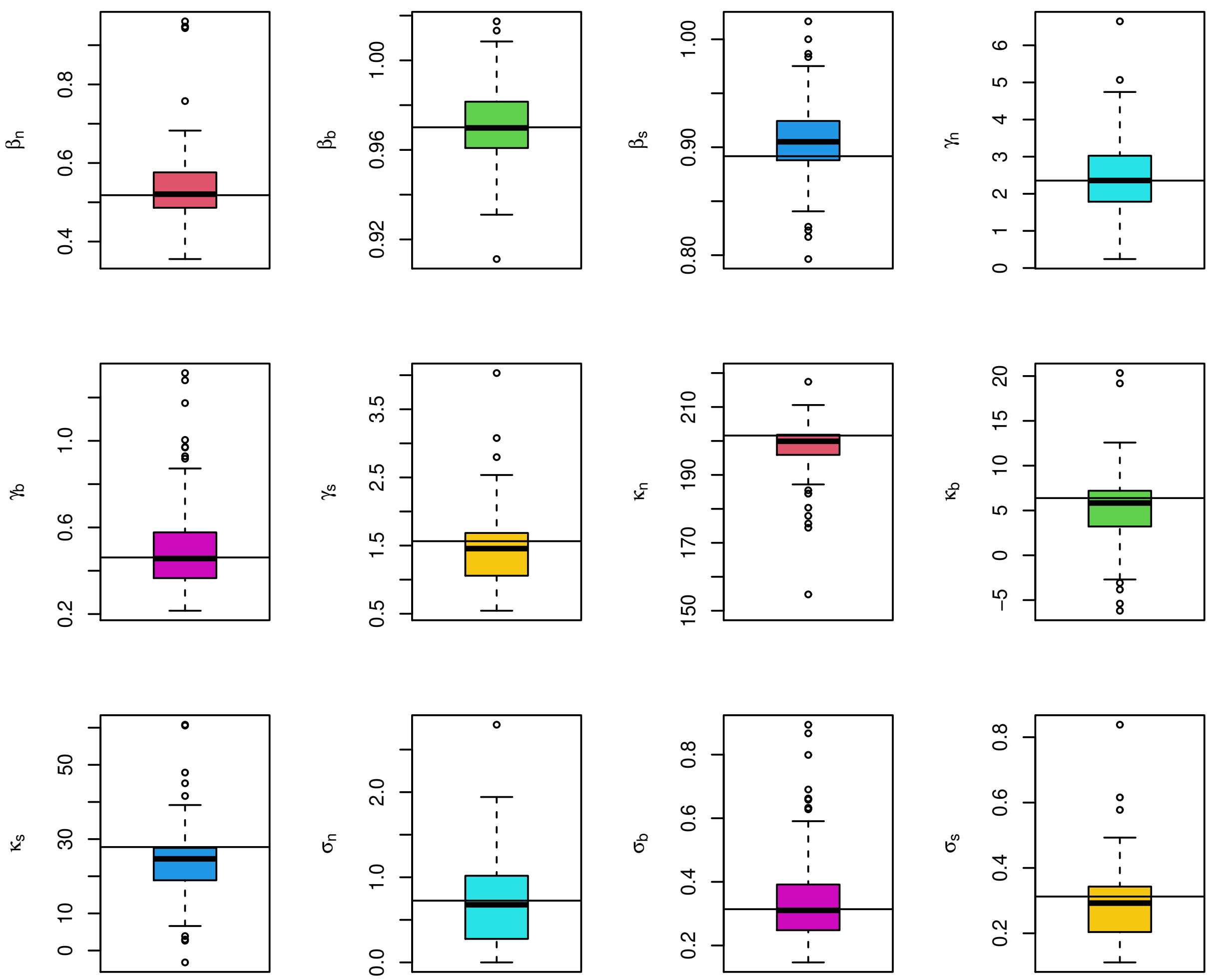
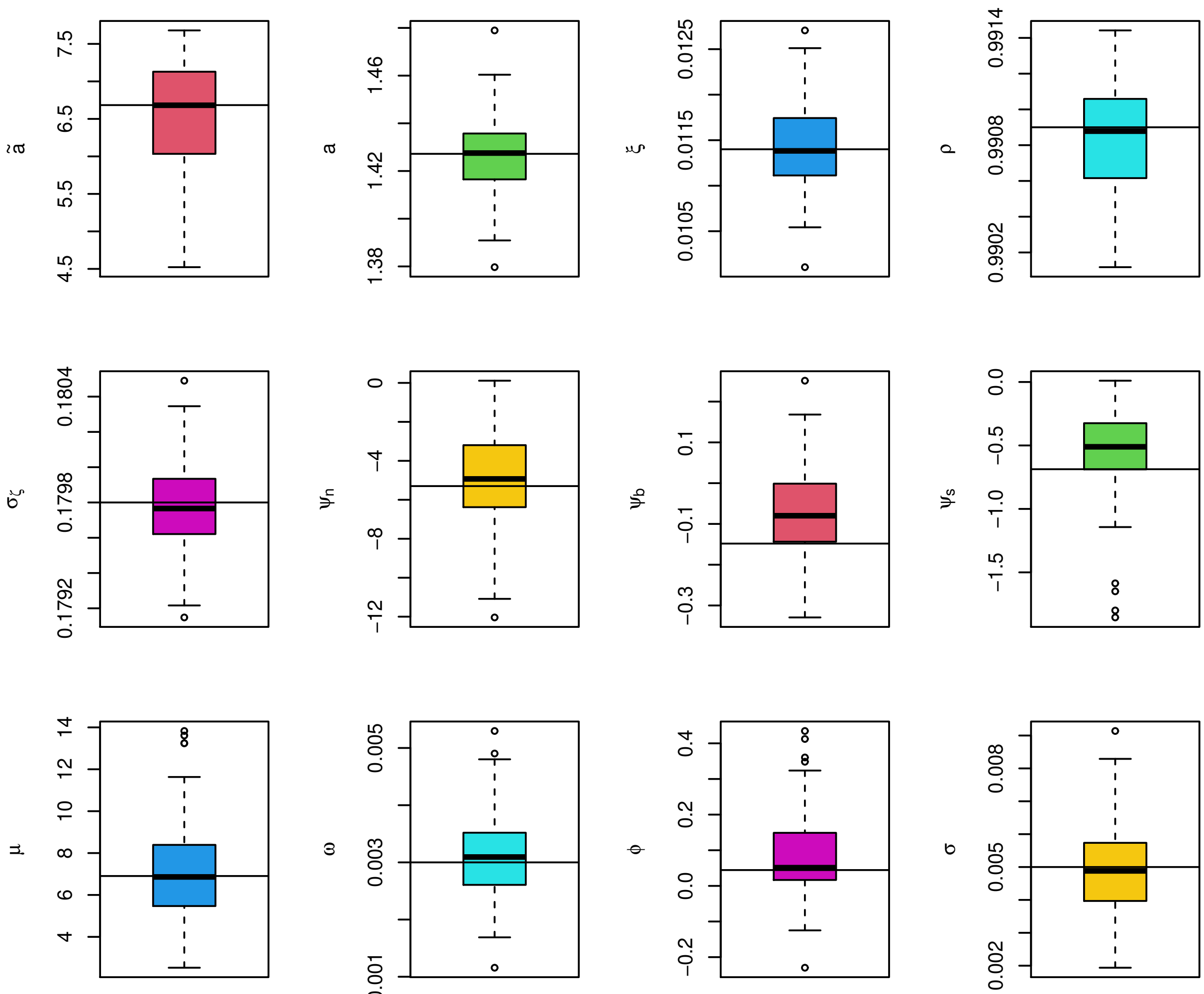


Figure 2: **Boxplots of 100 MC estimates of the structural parameters $\tilde{a}$, a, ξ, ρ, σ_ζ, ψ_n, ψ_b, ψ_s, μ, ω, ϕ, and σ. True parameters are reported with horizontal lines.**



are reported with horizontal lines. These figures show that all parameters are correctly estimated since true values are covered by the intervals given by the whiskers of the boxplots. Most of the 24 parameters are very precisely estimated since the true values are inside the boxes. The only exceptions are κ_s, ψ_b, and ψ_s, which are less precisely estimated than the other parameters. To check whether the parameters are consistently estimated, we report in Table 1 the Monte Carlo coverages (95%-COVERAGE) of the 95%-confidence intervals, obtained with the 100 Monte Carlo estimates plotted in the boxplots[2]. Under each coverage we report the two-sided p-values in parentheses. This Table shows that most parameters, including ψ_b and ψ_s, are accurately estimated as the 95%-coverages are not significantly different from the 95% at the 5% significance level. The only two parameters that provide coverages that are significantly different from 95% at the 5% level (p-values equal to 0.0314) are κ_s and ϕ.

4 Robustness checks

We assess the sensitivity of our empirical results with respect to the choice of the number of pseudo-datasets M used in indirect inference estimation. We also investigate the sensitivity of the results with respect to measurement errors and with respect to the choice of distributional assumptions in our limited participation model.

4.1 Alternative number of pseudo-datasets

According to Gouriéroux et al. (1993) asymptotic standard errors are multiplied by factor $\sqrt{1+M^{-1}}$ (as asymptotic variance is multiplied by $(1+M^{-1})$). The choice of $M=5$ ensures that the loss in standard deviation due to simulations is less than 10% ($\sqrt{1+5^{-1}} = 1.0954 < 1.1$). We run the empirical estimation with various numbers of pseudo-datasets $M = 4, 5, 6$ and report in Table 2 the parameter estimates. We report in the bottom row the proportion of correctly identified stockholders $\hat{\pi}_S$ for each M, along with 2-sided p-values (in

II estimates that are within 1.5 times the interquartile range from the first and third quartiles. II estimates that are outside the whiskers are drawn by dots.

[2] 95%-COVERAGE is calculated as the average number of times, across the Monte Carlo trials, that θ_j^0, i.e., the true value of the jth parameter, is contained in the univariate confidence interval $\hat{\theta}_j^i \pm \hat{\sigma}_j 1.96$, where $\hat{\sigma}_j$ is the standard deviation for the jth parameter over the 100 Monte Carlo replications and $\hat{\theta}_j^i$ is the estimator of the jth parameter in the ith Monte Carlo trial.

Table 1: **Coverage probabilities of 95% confidence intervals**

Symbol	95%-COVERAGE
β_n	0.95 (1.0000)
β_b	0.93 (0.4918)
β_s	0.93 (0.4918)
γ_n	0.94 (0.8200)
γ_b	0.92 (0.2462)
γ_s	0.97 (0.4918)
κ_n	0.96 (0.8200)
κ_b	0.93 (0.4918)
κ_s	0.90* (0.0341)
σ_n	0.97 (0.4918)
σ_b	0.92 (0.2462)
σ_s	0.97 (0.4918)
$\tilde{a}$	0.94 (0.8200)
a	0.93 (0.4918)
ξ	0.95 (1.0000)
ρ	0.97 (0.4918)
σ_ζ	0.95 (1.0000)
ψ_n	0.94 (0.8200)
ψ_b	0.93 (0.4918)
ψ_s	0.94 (0.8200)
μ	0.95 (1.0000)
ω	0.95 (1.0000)
ϕ	0.90* (0.0341)
σ	0.96 (0.8200)

parentheses) associated with the two-sample z test for proportions comparing $\hat{\pi}_S$ of $M = 4$ or $M = 6$ to the $M = 5$ case. This table shows that estimation results are essentially the same as $\hat{\pi}_S$ values are not significantly different from the $M = 5$ case and we could have used a simulation number of $M = 4$ for instance.

Table 2: **Empirical estimates for various number of simulated pseudo-data series.**

Symbol	Number of pseudo-datasets M		
	4	5	6
β_n	0.5247	0.5181	0.5239
β_b	0.9461	0.9701	0.9587
β_s	0.8032	0.8917	0.9077
γ_n	3.5253	2.3552	2.1366
γ_b	0.6740	0.4614	0.4201
γ_s	3.0990	1.5709	1.5839
κ_n	196.5766	201.5889	206.0849
κ_b	13.2834	6.3829	9.1954
κ_s	54.6875	27.8652	23.3907
σ_n	1.3752	0.7255	0.5544
σ_b	0.4617	0.3141	0.2864
σ_s	0.6218	0.3125	0.3183
$\tilde{a}$	7.0483	6.6840	6.2231
a	1.4345	1.4272	1.4305
ξ	0.0115	0.0114	0.0114
ρ	0.9909	0.9909	0.9909
σ_ζ	0.1799	0.1798	0.1798
ψ_n	−4.4986	−5.2972	−5.9781
ψ_b	−0.3253	−0.1482	−0.2564
ψ_s	−1.2669	−0.6872	−0.6225
μ	6.5087	6.9049	7.4469
ω	0.0031	0.0030	0.0030
ϕ	0.0322	0.0444	0.0443
σ	0.0046	0.0050	0.0053
$\hat{\pi}_S$	93.60% (0.2191)	93.31%	93.19% (0.2334)

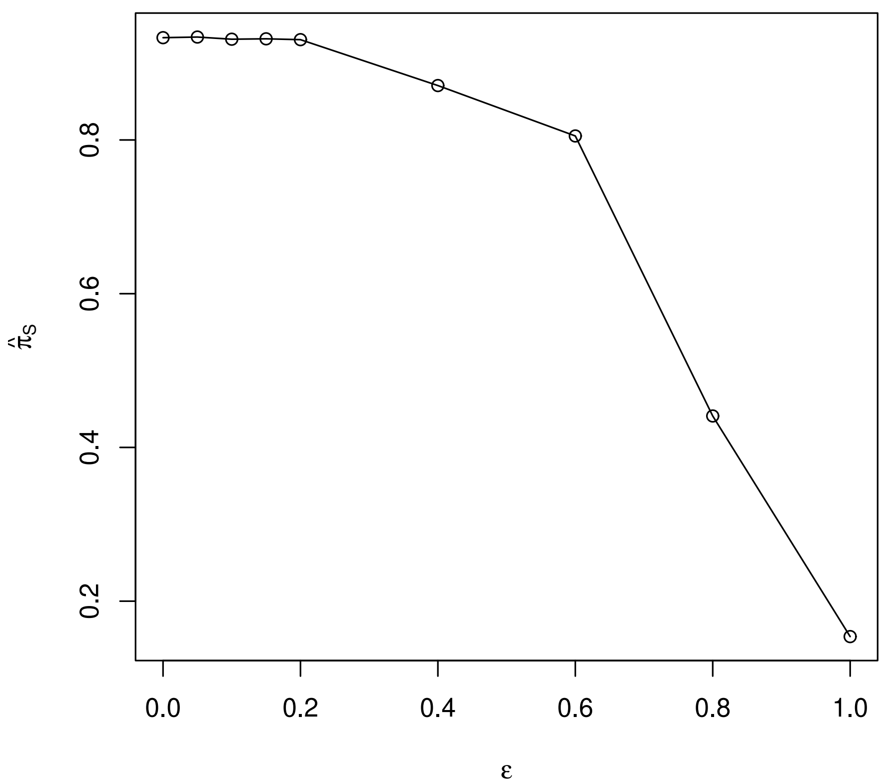
4.2 Measurement errors

We include measurement errors in the following way. For each household i, we replace the individual standard normally distributed increments in the log consumption growth and income processes with Student t_3 contaminated distributions of the form: $(1-\epsilon)\,\mathcal{N}(0,1)+\epsilon\, t_3$ with $\epsilon \in [0,1]$. Concretely, the originally standard normal increments ε^i_{t+1} are replaced with

$$(1 - B^i_{t+1})\varepsilon^i_{t+1} + B^i_{t+1}\upsilon^i_{t+1}$$

where $\{B^i_{t+1}\}$ are independent identically distributed Bernoulli variables with $\mathbb{P}(B^i_{t+1} = 1) = \epsilon$ and υ^i_{t+1} are Student t_3 variables. We use the same contamination for the z^i_t variables in the revenue. The parameter estimates are reported in Table 3 below.

Figure 3: **Proportion of correctly identified stockholders $\hat{\pi}_S$ for various contaminations ϵ.**



We report in the bottom row and in Figure 4.2 the proportion of correctly identified stockholders $\hat{\pi}_S$ for various contaminations ϵ, along with 2-sided p-values (in parentheses) associated with the two-sample z test for proportions comparing $\hat{\pi}_S$ of each contaminated

Table 3: **Empirical estimates for various types of increments**

Symbol	Contamination ϵ								
	0	0.05	0.10	0.15	0.20	0.40	0.60	0.80	1.00
β_n	0.5181	0.5307	0.5321	0.5090	0.5055	0.5962	0.5446	0.5317	0.5386
β_b	0.9701	0.9690	0.9740	0.9773	0.9753	0.9596	0.9700	0.9777	0.9769
β_s	0.8917	0.8976	0.8863	0.9029	0.8924	0.8962	0.8850	0.8797	0.8832
γ_n	2.3552	2.2257	2.2496	2.6308	2.5521	1.8405	2.0580	2.2354	2.2024
γ_b	0.4614	0.4772	0.4595	0.5144	0.4919	0.5537	0.4531	0.5567	0.5997
γ_s	1.5641	1.4889	1.5423	1.4110	1.4779	1.5731	2.0035	1.8096	1.8978
κ_n	201.5889	194.9222	193.1515	205.1788	208.6552	158.2293	188.8353	195.4955	192.3885
κ_b	6.3829	6.4930	5.1716	4.4850	4.8390	8.6723	6.3534	3.6858	4.0062
κ_s	27.8652	26.3135	29.5321	24.7952	27.9746	26.9975	30.0415	32.6101	31.5191
σ_n	0.7255	0.6661	0.6689	0.8297	0.7683	0.4957	0.4811	0.5152	0.4803
σ_b	0.3141	0.3240	0.3032	0.3322	0.3099	0.3173	0.2357	0.2570	0.2508
σ_s	0.3125	0.2945	0.2995	0.2758	0.2842	0.2978	0.3733	0.3141	0.3263
$\tilde{a}$	6.6840	6.5855	6.3534	6.5753	6.5633	6.0937	5.8873	5.6309	5.6364
a	1.4272	1.4320	1.4227	1.4180	1.4054	1.3658	1.3159	1.2012	1.1434
ξ	0.0114	0.0115	0.0115	0.0114	0.0112	0.0114	0.0114	0.0113	0.0113
ρ	0.9909	0.9909	0.9908	0.9909	0.9910	0.9909	0.9909	0.9909	0.9909
σ_ζ	0.1798	0.1787	0.1759	0.1739	0.1720	0.1642	0.1569	0.1496	0.1425
ψ_n	-5.2972	-5.1434	-5.0876	-5.3488	-5.4292	-4.1537	-5.0501	-5.2241	-5.1732
ψ_b	-0.1482	-0.1468	-0.1081	-0.0901	-0.0842	-0.2060	-0.1447	-0.0488	-0.0822
ψ_s	-0.6872	-0.6495	-0.7437	-0.6087	-0.7140	-0.6713	-0.7083	-0.8180	-0.7440
μ	6.9049	6.9044	6.9036	6.9042	6.9058	6.9052	6.9033	6.9030	6.9038
ω	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030	0.0030
ϕ	0.0444	0.0453	0.0455	0.0455	0.0415	0.0437	0.0471	0.0473	0.0468
σ	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050	0.0050
$\hat{\pi}_S$	93.39%	93.38% (0.4854)	93.22% (0.2446)	93.24% (0.2923)	93.14% (0.1819)	87.33% (0.0000)	80.86% (0.0000)	42.32% (0.0000)	14.78% (0.0000)

($\epsilon > 0$) case to the uncontaminated ($\epsilon = 0$) case. The parameter estimates are very similar and accuracies of the estimated stock market participations $\hat{\pi}_S$ are good and similar for contaminations up to $\epsilon \leq 0.2$, and $\hat{\pi}_S$ deteriorates substantially beyond this value. This means that our estimation method can withstand measurement errors up to 20%. This figure also shows that the Gaussian distribution is a legitimate assumption for the errors. Considering contaminated Gaussian errors does not improve the accuracy of uncovered stock participation, it actually significantly deteriorates the performance when contamination rate is 40% or beyond.

4.3 Alternative specifications

The volatility process $\tilde{\omega}_t$ in (2.3) is not necessarily positive, and needs to be modified in order to enter the square root in (2.2). Equation (2.4) is based on a smooth and positive transformations of $\tilde{\omega}_t$. We tried several alternatives of the transformation in (2.4). Concretely, in equation (2.4), we replaced $\omega^2/(2\omega - \tilde{\omega}_t)$ with two alternative functions $f(\tilde{\omega}_t)$ that are first order equivalent to (2.4) at $\tilde{\omega}_t = \omega$, i.e. satisfying $f(\omega) = \omega$, $\lim_{\tilde{\omega}_t \to -\infty} f(\tilde{\omega}_t) = 0$ and $\partial f / \partial \tilde{\omega}_t|_{\tilde{\omega}_t = \omega} = 1$. These two alternatives are

(a) $\omega \exp\left(\frac{\tilde{\omega}_t}{\omega} - 1\right)$ (exponential);

(b) $2\,\frac{\omega}{\pi}\left[\tan^{-1}\left(\frac{\pi\tilde{\omega}_t}{2\omega} - \frac{\pi}{2}\right) + \frac{\pi}{2}\right]$ (inverse tangent).

The first column in Table 4 reports the original estimates and $\hat{\pi}_S$ in Table 1 of the paper, and the second and third columns report the parameter estimates and $\hat{\pi}_S$ for the two alternative transformations. The two alternative transformations do not provide significantly better $\hat{\pi}_S$ results than the original specification reported in the first column.

Functions determining financial market participations in equations (2.5), respectively (2.6), are smooth and increasing functions in the revenue, respectively realized stock returns. As a robustness check, we tried several alternatives and replaced the functions $(1 - e^{-x})^2$ in (2.5) and (2.6) with two alternative functions $g(x)$ that are first order equivalent with $(1 - e^{-x})^2$ at $x = 0$, i.e. satisfying $g(0) = 0$, $\lim_{x \to \infty} g(x) = 1$ and $\partial g / \partial x|_{x=0} = 0$. These two alternatives are

(a) $\frac{x^2}{1+x^2}$ (polynomial);

(b) $\frac{2}{\pi}\tan^{-1}(x^2)$ (inverse tangent).

The fourth and fifth columns report the parameter estimates and $\hat{\pi}_S$ values for these two alternative transformations. The polynomial transformation in the fourth column does not provide significantly different $\hat{\pi}_S$ results from the one reported in the first column, while the inverse tangent variant in the fifth column provides significantly worse $\hat{\pi}_S$ results than the original specification.

Table 4: **Empirical estimates for alternative volatility and participation specifications**

	(2.4)-(2.6)	(2.4) Variants		(2.5)-(2.6) Variants	
Symbol	Unchanged	Exponential	Inverse tangent	Polynomial	Inverse tangent
β_n	0.5181	0.5679	0.5564	0.5897	0.5049
β_b	0.9701	0.9763	0.9781	0.9812	0.9679
β_s	0.8917	0.8710	0.8727	0.8883	0.8954
γ_n	2.3552	2.5720	2.5334	1.9755	2.4621
γ_b	0.4614	0.4522	0.4570	0.4484	0.4581
γ_s	1.5641	1.8205	1.7598	1.5617	1.5879
κ_n	201.5889	167.7987	174.7603	161.0874	209.2100
κ_b	6.3829	5.1132	4.5470	3.3234	6.8251
κ_s	27.8652	33.2239	32.9190	28.8584	26.7052
σ_n	0.7255	0.9371	0.8971	0.6347	0.7630
σ_b	0.3141	0.3108	0.3143	0.3104	0.3142
σ_s	0.3125	0.3612	0.3477	0.3078	0.3157
$\tilde{a}$	6.6840	7.0600	7.0173	5.9944	5.4314
a	1.4272	1.4347	1.4350	1.1873	1.1304
ξ	0.0114	0.0114	0.0114	0.0114	0.0114
ρ	0.9909	0.9909	0.9909	0.9909	0.9909
σ_ζ	0.1798	0.1798	0.1798	0.1798	0.1798
ψ_n	-5.2972	-4.1590	-4.4144	-4.2038	-5.4937
ψ_b	-0.1482	-0.1095	-0.0916	-0.0627	-0.1626
ψ_s	-0.6872	-0.8031	-0.8093	-0.7201	-0.6516
μ	6.9049	7.0759	6.9835	6.9060	6.9110
ω	0.0030	0.0034	0.0034	0.0030	0.0030
ϕ	0.0444	0.0661	0.0584	0.0445	0.0417
σ	0.0050	0.0045	0.0046	0.0050	0.0050
$\hat{\pi}_S$	93.39%	93.61% (0.2083)	93.59% (0.2302)	93.24% (0.2923)	90.08% (0.0000)

5 Lettau et al. (2019)-type regression with pricing kernel for $H = 1$

We redid the exercise in Table 5 of the paper using the pricing kernel and $H = 1$. Let us define

$$S_{i,t} = \beta_s e^{-\gamma_s \Delta \log c^i_{s,t+1}}$$

which is the pricing kernel of agent i (who is a stockholder at date t) at horizon $H = 1$.

We define $\overline{S}_t$ the average pricing kernel for stockholders:

$$\overline{S}_t = \frac{\sum_i 1_{i \in \{s\}} S_{i,t}}{\sum_i 1_{i \in \{s\}}}$$

where $1_{i \in \{s\}} = 1$ if i is a stockholder at date t (i.e. if $\tilde{p}^i_t p^i_t > 0.5$) and 0 otherwise.

Regressions become, for each set of portfolios j (Size/BM, REV, Size/OP and Size/INV):

$$R^e_{j,t} = \alpha^j_S + \beta^j_S \overline{S}_t + \epsilon_t \,,$$
$$\mathbb{E}[R^j_t - R^f_t] = \alpha_R + \beta_R \hat{\beta}^j_S + \upsilon_j,$$

where the loading factor on $\overline{S}_t$ is denoted by β^j_S. Table 5 reports the regression results. Intercept and slope coefficients are multiplied by 100 just like in Table 5 of the paper.

These results complete our long-run analysis and are convincingly satisfactory for most portfolios, with the exception of the Size/BM portfolio. The prices of risk are slightly larger than with the long-run factors and the R^2 are also quite large. Finally, the $\frac{RMSE}{RMSR}$ ratios are of comparable magnitude to those in the long-run analysis.

In our initial exercise, we regress the excess return on our macroeconomic factor to obtain our risk loading. We then regress the portfolio returns on this risk loading. The results are overall better than with the pricing kernel as a risk factor (larger R^2 and significant β). Compared to the results of Lettau et al. (2019), the R^2 are admittedly overall smaller (even though not small), but we rely on income data that are of much poorer quality that those of Lettau et al. (2019), who use very reliable tax data. This is especially true at the top of the income distribution that matters for stock holdings. However, as in our case, their α are significantly different from zero, which means that their model, like ours, is partly misspecified and does not fully capture the price of risk in the cross-section.

Overall, we see these two exercises as supporting evidence that our consumption-based asset pricing model – with few twists compared to the standard model – is able to capture a non-negligible share of the price of risk in the cross-section.

Table 5: **OLS regressions for Fama-McBeth analysis with characteristic-based portfolios**

	Equity portfolios	
	Panel A: Size/BM	Panel B: REV
$\hat{\alpha}_R$	2.56 [2.33,2.80]	2.77 [2.68,2.86]
$\hat{\beta}_R$	1.27 [−8.17,10.78]	8.32 [5.38,11.29]
R^2	0.00 [0.00,0.21]	0.79 [0.55,0.94]
$\frac{RMSE}{RMSR}$	0.22	0.05
	Panel C: Size/INV	Panel D: Size/OP
$\hat{\alpha}_R$	2.70 [2.51,2.90]	2.63 [2.45,2.81]
$\hat{\beta}_R$	12.32 [−1.78,26.55]	21.39 [7.97,34.76]
R^2	0.11 [0.00,0.41]	0.30 [0.05,0.59]
$\frac{RMSE}{RMSR}$	0.18	0.17

Table 6: This table reports OLS regression estimates of the models $\mathbb{E}[R_t^j - R_t^f] = \alpha_R + \beta_R \hat{\beta}_S^j + \upsilon_j$, where $\{\hat{\beta}_S^j\}$ are obtained from OLS regressions of the models $R_{j,t}^e = \alpha_S^j + \beta_S^j \overline{S}_t + \epsilon_t$ for Reversal Fama stocks, with pricing kernel $\overline{S}_t$s and $R_{j,t}^e = R_{j,t} - R_t^f$. Bootstrapped 95% confidence intervals are reported under regression values. Intercept and slope coefficients are multiplied by 100.

6 Estimating the distribution of ω_t conditional on $l_{1:t} \equiv \{\log R_{t'}^s - \log R_{t'}^f\}_{t'=1,\ldots,t}$ using a particle filter

At each date t, the equity premium $l_t = \log R_t^s - \log R_t^f$, $t = 1, \ldots, T$ is assumed to be generated from the equity premium process with hidden states ω_t. The probability density function of the states ω_t given the observations $l_{1:t}$ is not available in closed form but can be easily obtained via a bootstrap particle filter (Gordon, Salmond and Smith 1993) as described below.

At date $t = 0$, generate particles $\{\tilde{\omega}_0^{(j)}\}_{j=1}^J$ using the stationary distribution $\mathcal{N}(\omega, \frac{\sigma^2}{1-\phi^2})$, where J is a fixed large positive integer. For $t \geq 1$, iterate the following three steps.

Step 1 (Sampling): For every $j = 1, \dots, J$, simulate the filter forward as follows. Generate $\tilde{\omega}_t^{(j,*)}$ using $\tilde{\omega}_{t-1}^{(j)}$ by drawing $\tilde{\omega}_t^{(j,*)}$ from $\mathcal{N}[\omega + \phi(\tilde{\omega}_{t-1}^{(j)} - \omega), \sigma^2]$. For each $j = 1, \dots, J$, calculate $\omega_t^{(j,*)} = \tilde{\omega}_t^{(j,*)}$ if $\tilde{\omega}_t^{(j,*)} \geq \omega$ and $\omega_t^{(j,*)} = \omega/(2\omega - \tilde{\omega}_t^{(j,*)})$ otherwise.

Step 2 (Correction): Given the new equity premium l_t, compute:

$$q_t^{(j)} = f(l_t|\omega_t^{(j,*)}) = \frac{1}{\sqrt{2\pi\omega_t^{(j,*)}}} \exp\left[-\frac{(l_t - \mu\,\omega_t^{(j,*)})^2}{2\omega_t^{(j,*)}}\right], \quad j = 1, \dots, J\,,$$

Step 3 (Selection): For each $j = 1, \dots, J$, draw $\tilde{\omega}_t^{(j)}$ from $\tilde{\omega}_t^{(1,*)}, \dots, \tilde{\omega}_t^{(J,*)}$ with importance weights $r_t^{(1)}, \dots, r_t^{(J)}$, where $r_t^{(j)} = q_t^{(j)} / \sum_{j'=1}^J q_t^{(j')}$. For each $j = 1, \dots, J$,, calculate $\omega_t^{(j)} = \tilde{\omega}_t^{(j)}$ if $\tilde{\omega}_t^{(j)} \geq \omega$ and $\omega_t^{(j)} = \omega/(2\omega - \tilde{\omega}_t^{(j)})$ otherwise.

At each date t, the set $\{\omega_t^{(j)}\}_{j=1}^J$ finitely estimates the conditional distribution of ω_t given $l_{1:t}$.

7 Estimating the distribution of ω_{t+1} conditional on $l_{1:t} \equiv \{\log R_{t'}^s - \log R_{t'}^f\}_{t'=1,\dots,t}$ using a particle filter

To estimate the distribution of ω_{t+1} using equity premium observations until date t, simulate a set of particles $\{\tilde{\omega}_t^{(j)}\}_{j=1}^J$ following the algorithm described in Section 6. Then, generate $\tilde{\omega}_{t+1}^{(j,*)}$ using $\tilde{\omega}_t^{(j)}$ by drawing $\tilde{\omega}_{t+1}^{(j,*)}$ from $\mathcal{N}[\omega + \phi(\tilde{\omega}_t^{(j)} - \omega), \sigma^2]$. For each $j = 1, \dots, J$, calculate $\omega_{t+1}^{(j,*)} = \tilde{\omega}_{t+1}^{(j,*)}$ if $\tilde{\omega}_{t+1}^{(j,*)} \geq \omega$ and $\omega_{t+1}^{(j,*)} = \omega/(2\omega - \tilde{\omega}_{t+1}^{(j,*)})$ otherwise.

The set $\{\omega_{t+1}^{(j,*)}\}$ finitely estimates the conditional distribution of ω_{t+1} given $l_{1:t}$.

8 Estimating the distribution of $\omega_{1:T}$ conditional on $l_{1:T} \equiv \{\log R_{t'}^s - \log R_{t'}^f\}_{t'=1,\dots,T}$ using a particle smoother

The probability density function of the states ω_t given the observations $l_{1:T}$ is not available in closed form but can easily be obtained via a particle smoother (Godsill et al. 2004) as described below.

Step 1 (Particle filtering): Use the particle filter defined in Section 6 to obtain an approximate particle representation of $f(\tilde{\omega}_t|l_{1:t})$ at each date $t = 1, \ldots, T$. Denote these particles by $\{\tilde{\omega}_t^{(j)}\}_{t=1,\ldots,T}^{j=1,\ldots,J}$.

For $k = 1, \ldots, K$, replicate Steps 2-4, where K is a fixed large positive integer.

Step 2 (Positioning of the backward simulation): Choose $\tilde{\omega}_T^{(k,*)} = \tilde{\omega}_T^{(j)}$ with probability $1/J$.

Step 3 (Backward simulation): For $t = T-1, \ldots, 1$ and each $j = 1, \ldots, J$

(i) compute the importance weights

$$q_{t|t+1}^{(j,k)} = f(\tilde{\omega}_{t+1}^{(k,*)}|\tilde{\omega}_t^{(j)}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{[\tilde{\omega}_{t+1}^{(k,*)} - (\omega + \phi(\tilde{\omega}_t^{(j)} - \omega)]^2}{2\sigma^2}\right\}, \quad j = 1, \ldots, J\,;$$

(ii) choose $\tilde{\omega}_t^{(k,*)} = \tilde{\omega}_t^{(j)}$ with probability $r_{t|t+1}^{(j,k)}$, where $r_{t|t+1}^{(j,k)} = q_{t|t+1}^{(j,k)}/\sum_{j'=1}^{J} q_{t|t+1}^{(j',k)}$.

Step 4 (Path drawing): $\tilde{\omega}_{1:T}^{(k,*)} = (\tilde{\omega}_1^{(k,*)}, \ldots, \tilde{\omega}_T^{(k,*)})$ is an approximate realization from $f(\tilde{\omega}_{1:T}|l_{1:T})$.

For each $k = 1, \ldots, K$, and $t = 1, \ldots, T$, calculate $\omega_t^{(k,*)} = \tilde{\omega}_t^{(k,*)}$ if $\tilde{\omega}_t^{(k,*)} \geq \omega$ and $\omega_t^{(k,*)} = \omega/(2\omega - \tilde{\omega}_t^{(k,*)})$ otherwise. For each $k = 1, \ldots, K$, $\omega_{1:T}^{(k,*)} = (\omega_1^{(k,*)}, \ldots, \omega_T^{(k,*)})$ is an approximate realization from $f(\omega_{1:T}|l_{1:T})$.

9 Simulated maximum likelihood estimation of μ, ω, ϕ and σ using a particle filter

We can estimate μ, ω, ϕ, and σ by maximizing the simulated log-likelihood function associated with the equity premium process:

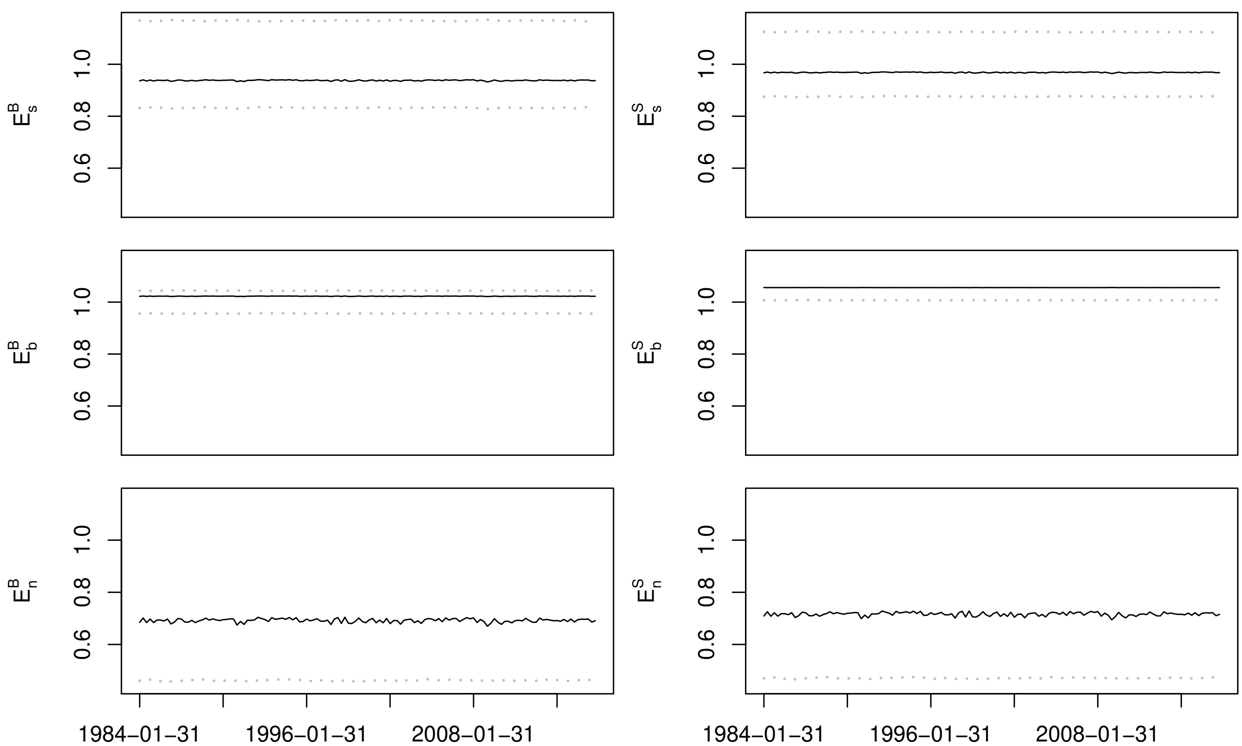
$$\sum_{t=1}^{T} \log\left(\frac{1}{J}\sum_{j=1}^{J} q_t^{(j)}\right),$$

where $q_t^{(j)}$ are defined in the second step of the particle filtering algorithm in Section 6. In order to obtain a smoother objective function, we choose a large number of particles $J = 10^6$.

10 Euler tests

In Figure 4 (continuous lines), we report the sample means $\{\overline{E}^y_{x,t}\}_{t=1}^T\}$ obtained using the J particles $\{\omega_t^j\}_{j=1}^J$ for $x = n, b, s$ and $y = B, S$. In addition, we also plot (dotted lines) the 5% critical values for $\{\overline{E}^y_{x,t}\}_{t=1}^T\}$ associated with the tests (T1)–(T6). Bond Euler conditions are in the left panels, while stock equations are in the right panels.

Figure 4: **Test statistics (continuous lines) and associated critical values (dotted lines) for the six Euler conditions.**



For bonds, the quantities $\{E^B_{s,t}\}_{t=1,\ldots,T}$ (top left panel) and $\{E^B_{b,t}\}_{t=1,\ldots,T}$ (middle left panel) lie within the Euler acceptance regions and (T1) and (T2) hold for all t. In the bottom left panel, we see that $E^B_{n,t}$ is less than one but not significantly for all t. Note that the test in the bottom left panel is unilateral and not bilateral as it was in the top panels. We can therefore conclude that (T3) holds – though not significantly – at all dates t.

For stocks, $E^S_{s,t}$ (top right panel) is not significantly different from 1 and (T4) holds for all t. In the middle right panel, we see that $E^S_{b,t}$ is significantly greater than one and that (T5) holds for all t, thereby confirming the presence of a stock market participation cost. In the bottom right panel, $E^S_{n,t}$ is less than one, but not significantly, and (T6) holds, but not significantly, for all t. One conclusion of the tests in Figure 4 is that our estimation is consistent with our initial interpretation of household types: stockholders (who also hold bonds) are of type s, bondholders (who do not hold stocks) are of type b, and nonparticipants are of type n.

11 Unconditional Euler tests

We test whether the Euler conditions (T1)-(T6) in Section 4.3 of the main paper are satisfied for equity return data available from Kenneth French's Dartmouth website on 25 size/book-to-market sorted portfolios (Size/BM, Table 7), 10 long-run reversal portfolios (REV, Table 10), 25 size/investment portfolios (Size/INV, Table 8), 25 size/operating profitability portfolios (Size/OP, Table 9) and 10 industry portfolios (IND, Table 11). For each of these portfolios, we estimate the four equity premium parameters μ, ω, ϕ, and σ driving the equity premium dynamics. The estimation is performed using the simulated maximum likelihood described in Section 7 of the appendix. In Tables 10-11, we report the sample means and associated standard errors of $\{E^B_{x,t}\}_{t=1,\ldots,T}$ for bonds and $\{E^S_{x,t}\}_{t=1,\ldots,T}$ for stocks. The results are clear and strongly support the limited participation model at the 5% significance level. For bond holding, the three Euler conditions hold. For stock holding, the Euler condition for nonparticipants is significantly less than one most of the time. For stockholders, E^S_s is not significantly different from one most of the time. For bondholders, E^S_b is greater than one for most portfolios but not significantly. Overall, the tests for Euler conditions are consistent with the model and confirm that our estimated model properly isolates three categories of households: stockholders, bondholders, and nonparticipants.

Table 7: **Unconditional Euler tests for Size/BM**

	Euler Bond Conditions			Euler Stock Conditions		
Portfolio	$\overline{E}_n^B$	$\overline{E}_b^B$	$\overline{E}_s^B$	$\overline{E}_n^S$	$\overline{E}_b^S$	$\overline{E}_s^S$
SMALL LoBM	0.1314	0.8963	0.5810	0.1373	0.9038	0.5902
	(0.1706)	(0.1102)	(0.2520)	(0.1757)	(0.1053)	(0.2497)
ME1 BM2	0.2388	0.9490	0.7049	0.2486	0.9725	0.7241
	(0.2493)	(0.0752)	(0.2113)	(0.2550)	(0.0587)	(0.2031)
ME1 BM3	0.2572	0.9745	0.7692	0.2697	0.9976	0.7904
	(0.2181)	(0.0447)	(0.1428)	(0.2231)	(0.0315)	(0.1342)
ME1 BM4	0.3159	0.9812	0.7928	0.3310	1.0152	0.8217
	(0.2606)	(0.0458)	(0.1487)	(0.2656)	(0.0238)	(0.1350)
SMALL HiBM	0.2640	0.9662	0.7484	0.2765	0.9990	0.7752
	(0.2477)	(0.0577)	(0.1756)	(0.2535)	(0.0360)	(0.1636)
ME2 BM1	0.1964	0.9378	0.6736	0.2047	0.9543	0.6885
	(0.2171)	(0.0799)	(0.2156)	(0.2226)	(0.0689)	(0.2101)
ME2 BM2	0.3372	0.9757	0.7803	0.3500	1.0041	0.8040
	(0.2996)	(0.0571)	(0.1787)	(0.3043)	(0.0361)	(0.1662)
ME2 BM3	0.3744	0.9878	0.8156	0.3892	1.0158	0.8401
	(0.2969)	(0.0453)	(0.1498)	(0.3008)	(0.0255)	(0.1367)
ME2 BM4	0.3704	0.9904	0.8227	0.3862	1.0195	0.8485
	(0.2801)	(0.0405)	(0.1365)	(0.2839)	(0.0213)	(0.1233)
ME2 BM5	0.2799	0.9641	0.7453	0.2920	0.9968	0.7714
	(0.2682)	(0.0634)	(0.1898)	(0.2739)	(0.0402)	(0.1775)
ME3 BM1	0.2335	0.9553	0.7183	0.2433	0.9747	0.7356
	(0.2353)	(0.0660)	(0.1920)	(0.2407)	(0.0532)	(0.1848)
ME3 BM2	0.3717	0.9860	0.8104	0.3860	1.0136	0.8345
	(0.3015)	(0.0476)	(0.1560)	(0.3055)	(0.0277)	(0.1430)
ME3 BM3	0.3910	0.9970	0.8434	0.4079	1.0227	0.8672
	(0.2624)	(0.0328)	(0.1142)	(0.2653)	(0.0173)	(0.1028)
ME3 BM4	0.3852	0.9908	0.8248	0.4006	1.0186	0.8495
	(0.2943)	(0.0422)	(0.1417)	(0.2979)	(0.0229)	(0.1286)
ME3 BM5	0.3581	0.9823	0.7995	0.3727	1.0156	0.8273
	(0.3008)	(0.0509)	(0.1641)	(0.3053)	(0.0266)	(0.1491)
ME4 BM1	0.3310	0.9690	0.7628	0.3430	0.9980	0.7862
	(0.3091)	(0.0655)	(0.1980)	(0.3141)	(0.0430)	(0.1855)
ME4 BM2	0.4437	0.9955	0.8423	0.4583	1.0192	0.8637
	(0.3312)	(0.0429)	(0.1457)	(0.3336)	(0.0248)	(0.1328)
ME4 BM3	0.4092	0.9916	0.8289	0.4235	1.0158	0.8506
	(0.3154)	(0.0443)	(0.1485)	(0.3185)	(0.0266)	(0.1362)
ME4 BM4	0.4472	0.9984	0.8509	0.4631	1.0237	0.8738
	(0.3180)	(0.0387)	(0.1336)	(0.3202)	(0.0203)	(0.1203)
ME4 BM5	0.2799	0.9739	0.7698	0.2933	1.0048	0.7961
	(0.2454)	(0.0500)	(0.1578)	(0.2508)	(0.0304)	(0.1460)
BIG LoBM	0.4623	1.0029	0.8651	0.4790	1.0261	0.8869
	(0.3014)	(0.0330)	(0.1169)	(0.3030)	(0.0172)	(0.1048)
ME5 BM2	0.5455	1.0082	0.8855	0.5612	1.0292	0.9051
	(0.3496)	(0.0343)	(0.1228)	(0.3496)	(0.0175)	(0.1097)
ME5 BM3	0.5249	1.0094	0.8883	0.5435	1.0349	0.9121
	(0.3129)	(0.0294)	(0.1069)	(0.3130)	(0.0114)	(0.0927)
ME5 BM4	0.4281	0.9844	0.8109	0.4402	1.0078	0.8308
	(0.3612)	(0.0577)	(0.1844)	(0.3644)	(0.0378)	(0.1717)
BIG HiBM	0.3710	0.9756	0.7832	0.3834	1.0040	0.8064
	(0.3325)	(0.0624)	(0.1932)	(0.3368)	(0.0395)	(0.1798)

Table 8: **Unconditional Euler tests for Size/INV**

	Euler Bond Conditions			Euler Stock Conditions		
Portfolio	$\overline{E}_n^B$	$\overline{E}_b^B$	$\overline{E}_s^B$	$\overline{E}_n^S$	$\overline{E}_b^S$	$\overline{E}_s^S$
SMALL LoINV	0.1908	0.9287	0.6540	0.2007	0.9723	0.6835
	(0.2187)	(0.0896)	(0.2321)	(0.2257)	(0.0592)	(0.2212)
ME1 INV2	0.2807	0.9783	0.7819	0.2950	1.0090	0.8086
	(0.2318)	(0.0436)	(0.1412)	(0.2369)	(0.0256)	(0.1298)
ME1 INV3	0.3082	0.9882	0.8118	0.3251	1.0203	0.8407
	(0.2183)	(0.0335)	(0.1136)	(0.2227)	(0.0166)	(0.1020)
ME1 INV4	0.3371	0.9831	0.7998	0.3509	1.0089	0.8226
	(0.2763)	(0.0465)	(0.1515)	(0.2806)	(0.0292)	(0.1402)
SMALL HiINV	0.1831	0.9352	0.6657	0.1907	0.9460	0.6772
	(0.2051)	(0.0797)	(0.2136)	(0.2104)	(0.0728)	(0.2096)
ME2 INV1	0.2784	0.9560	0.7261	0.2896	0.9885	0.7509
	(0.2799)	(0.0739)	(0.2122)	(0.2858)	(0.0496)	(0.2003)
ME2 INV2	0.3883	0.9941	0.8347	0.4047	1.0220	0.8598
	(0.2793)	(0.0373)	(0.1277)	(0.2826)	(0.0192)	(0.1149)
ME2 INV3	0.3620	0.9944	0.8337	0.3792	1.0223	0.8594
	(0.2460)	(0.0326)	(0.1130)	(0.2494)	(0.0167)	(0.1014)
ME2 INV4	0.3751	0.9849	0.8079	0.3896	1.0153	0.8337
	(0.3089)	(0.0496)	(0.1615)	(0.3130)	(0.0271)	(0.1472)
ME2 INV5	0.2216	0.9389	0.6804	0.2300	0.9544	0.6945
	(0.2425)	(0.0843)	(0.2267)	(0.2479)	(0.0733)	(0.2211)
ME3 INV1	0.2973	0.9757	0.7763	0.3107	1.0048	0.8012
	(0.2589)	(0.0506)	(0.1602)	(0.2640)	(0.0315)	(0.1486)
ME3 INV2	0.4354	0.9994	0.8530	0.4520	1.0249	0.8764
	(0.2982)	(0.0354)	(0.1236)	(0.3005)	(0.0180)	(0.1108)
ME3 INV3	0.4247	1.0033	0.8644	0.4444	1.0321	0.8912
	(0.2490)	(0.0267)	(0.0958)	(0.2510)	(0.0106)	(0.0835)
ME3 INV4	0.3586	0.9891	0.8180	0.3742	1.0175	0.8433
	(0.2738)	(0.0408)	(0.1368)	(0.2778)	(0.0223)	(0.1243)
ME3 INV5	0.2596	0.9529	0.7165	0.2694	0.9743	0.7344
	(0.2655)	(0.0744)	(0.2117)	(0.2709)	(0.0590)	(0.2036)
ME4 INV1	0.3720	0.9844	0.8062	0.3858	1.0113	0.8295
	(0.3075)	(0.0499)	(0.1622)	(0.3115)	(0.0301)	(0.1494)
ME4 INV2	0.4199	1.0004	0.8552	0.4381	1.0286	0.8811
	(0.2712)	(0.0313)	(0.1106)	(0.2736)	(0.0140)	(0.0976)
ME4 INV3	0.4606	1.0024	0.8635	0.4773	1.0263	0.8858
	(0.3036)	(0.0337)	(0.1189)	(0.3053)	(0.0172)	(0.1064)
ME4 INV4	0.4654	0.9976	0.8498	0.4805	1.0231	0.8725
	(0.3416)	(0.0423)	(0.1447)	(0.3436)	(0.0223)	(0.1305)
ME4 INV5	0.2702	0.9512	0.7143	0.2802	0.9758	0.7339
	(0.2781)	(0.0786)	(0.2208)	(0.2837)	(0.0601)	(0.2117)
BIG LoINV	0.4428	1.0030	0.8645	0.4617	1.0315	0.8908
	(0.2760)	(0.0300)	(0.1068)	(0.2779)	(0.0122)	(0.0934)
ME5 INV2	0.5624	1.0161	0.9112	0.5807	1.0331	0.9283
	(0.2652)	(0.0206)	(0.0778)	(0.2642)	(0.0105)	(0.0689)
ME5 INV3	0.5803	1.0119	0.8985	0.5963	1.0318	0.9174
	(0.3516)	(0.0317)	(0.1151)	(0.3507)	(0.0156)	(0.1021)
ME5 INV4	0.5770	1.0080	0.8864	0.5915	1.0291	0.9057
	(0.3845)	(0.0383)	(0.1358)	(0.3839)	(0.0196)	(0.1214)
BIG HiINV	0.3729	0.9825	0.8013	0.3862	1.0090	0.8240
	(0.3149)	(0.0528)	(0.1698)	(0.3190)	(0.0328)	(0.1570)

Table 9: **Unconditional Euler tests for Size/OP**

Portfolio	Euler Bond Conditions $\overline{E}_n^B$	$\overline{E}_b^B$	$\overline{E}_s^B$	Euler Stock Conditions $\overline{E}_n^S$	$\overline{E}_b^S$	$\overline{E}_s^S$
SMALL LoOP	0.1704	0.9244	0.6414	0.1777	0.9385	0.6546
	(0.1997)	(0.0893)	(0.2286)	(0.2052)	(0.0801)	(0.2243)
ME1 OP2	0.2766	0.9839	0.7969	0.2920	1.0129	0.8233
	(0.2022)	(0.0343)	(0.1148)	(0.2068)	(0.0197)	(0.1049)
ME1 OP3	0.3300	0.9875	0.8113	0.3454	1.0153	0.8364
	(0.2492)	(0.0386)	(0.1294)	(0.2534)	(0.0219)	(0.1181)
ME1 OP4	0.2888	0.9815	0.7913	0.3034	1.0104	0.8170
	(0.2286)	(0.0404)	(0.1328)	(0.2334)	(0.0239)	(0.1220)
SMALL HiOP	0.2384	0.9554	0.7193	0.2489	0.9809	0.7404
	(0.2403)	(0.0669)	(0.1942)	(0.2460)	(0.0498)	(0.1853)
ME2 OP1	0.2163	0.9335	0.6687	0.2244	0.9492	0.6825
	(0.2416)	(0.0898)	(0.2356)	(0.2471)	(0.0786)	(0.2302)
ME2 OP2	0.3597	0.9822	0.7994	0.3732	1.0087	0.8222
	(0.3028)	(0.0513)	(0.1652)	(0.3070)	(0.0320)	(0.1529)
ME2 OP3	0.3627	0.9938	0.8320	0.3796	1.0216	0.8575
	(0.2509)	(0.0338)	(0.1164)	(0.2544)	(0.0175)	(0.1047)
ME2 OP4	0.3806	0.9858	0.8107	0.3951	1.0155	0.8361
	(0.3111)	(0.0491)	(0.1605)	(0.3151)	(0.0271)	(0.1463)
ME2 OP5	0.2671	0.9729	0.7659	0.2809	1.0068	0.7945
	(0.2343)	(0.0490)	(0.1544)	(0.2399)	(0.0282)	(0.1421)
ME3 OP1	0.2286	0.9351	0.6741	0.2370	0.9522	0.6886
	(0.2529)	(0.0906)	(0.2384)	(0.2584)	(0.0780)	(0.2325)
ME3 OP2	0.3984	0.9915	0.8279	0.4132	1.0169	0.8506
	(0.3048)	(0.0430)	(0.1444)	(0.3080)	(0.0249)	(0.1319)
ME3 OP3	0.3846	1.0007	0.8543	0.4029	1.0249	0.8775
	(0.2192)	(0.0249)	(0.0890)	(0.2216)	(0.0130)	(0.0797)
ME3 OP4	0.3582	0.9909	0.8234	0.3741	1.0183	0.8482
	(0.2634)	(0.0378)	(0.1280)	(0.2672)	(0.0207)	(0.1162)
ME3 OP5	0.3216	0.9804	0.7912	0.3362	1.0129	0.8188
	(0.2694)	(0.0478)	(0.1545)	(0.2744)	(0.0262)	(0.1410)
ME4 OP1	0.2582	0.9486	0.7070	0.2675	0.9678	0.7232
	(0.2693)	(0.0796)	(0.2217)	(0.2747)	(0.0655)	(0.2144)
ME4 OP2	0.3931	0.9876	0.8165	0.4072	1.0142	0.8397
	(0.3167)	(0.0482)	(0.1586)	(0.3203)	(0.0283)	(0.1455)
ME4 OP3	0.4514	1.0027	0.8640	0.4686	1.0267	0.8865
	(0.2898)	(0.0318)	(0.1128)	(0.2916)	(0.0161)	(0.1008)
ME4 OP4	0.4243	0.9965	0.8440	0.4404	1.0231	0.8679
	(0.3048)	(0.0386)	(0.1327)	(0.3075)	(0.0200)	(0.1194)
ME4 OP5	0.4102	0.9923	0.8310	0.4254	1.0198	0.8552
	(0.3131)	(0.0434)	(0.1459)	(0.3163)	(0.0233)	(0.1322)
BIG LoOP	0.3371	0.9579	0.7380	0.3462	0.9731	0.7515
	(0.3328)	(0.0813)	(0.2314)	(0.3372)	(0.0687)	(0.2241)
ME5 OP2	0.4626	0.9988	0.8529	0.4770	1.0191	0.8718
	(0.3319)	(0.0401)	(0.1382)	(0.3338)	(0.0247)	(0.1267)
ME5 OP3	0.4968	0.9978	0.8522	0.5102	1.0188	0.8711
	(0.3717)	(0.0460)	(0.1560)	(0.3732)	(0.0280)	(0.1430)
ME5 OP4	0.5989	1.0151	0.9096	0.6151	1.0325	0.9266
	(0.3334)	(0.0275)	(0.1018)	(0.3319)	(0.0142)	(0.0905)
BIG HiOP	0.4952	1.0086	0.8846	0.5130	1.0295	0.9047
	(0.2833)	(0.0269)	(0.0979)	(0.2839)	(0.0137)	(0.0872)

Table 10: **Unconditional Euler tests for REV**

	Euler Bond Conditions			Euler Stock Conditions		
Portfolio	$\overline{E}_n^B$	$\overline{E}_b^B$	$\overline{E}_s^B$	$\overline{E}_n^S$	$\overline{E}_b^S$	$\overline{E}_s^S$
Loprior	0.2799	0.9456	0.7050	0.2899	0.9742	0.7260
	(0.2886)	(0.0889)	(0.2396)	(0.2944)	(0.0660)	(0.2296)
Prior 2	0.3688	0.9885	0.8175	0.3839	1.0162	0.8419
	(0.2833)	(0.0437)	(0.1448)	(0.2873)	(0.0246)	(0.1321)
Prior 3	0.4717	1.0016	0.8617	0.4878	1.0256	0.8838
	(0.3225)	(0.0366)	(0.1280)	(0.3242)	(0.0189)	(0.1148)
Prior 4	0.5377	1.0049	0.8753	0.5525	1.0272	0.8956
	(0.3678)	(0.0390)	(0.1371)	(0.3682)	(0.0203)	(0.1230)
Prior 5	0.5140	1.0052	0.8748	0.5299	1.0278	0.8958
	(0.3409)	(0.0357)	(0.1265)	(0.3416)	(0.0182)	(0.1131)
Prior 6	0.5301	1.0130	0.9000	0.5519	1.0409	0.9264
	(0.2699)	(0.0228)	(0.0847)	(0.2695)	(0.0059)	(0.0711)
Prior 7	0.5264	1.0143	0.9041	0.5485	1.0398	0.9287
	(0.2391)	(0.0192)	(0.0723)	(0.2386)	(0.0058)	(0.0611)
Prior 8	0.5272	1.0136	0.9018	0.5479	1.0373	0.9247
	(0.2548)	(0.0210)	(0.0787)	(0.2544)	(0.0077)	(0.0675)
Prior 9	0.4783	1.0058	0.8750	0.4955	1.0277	0.8958
	(0.2931)	(0.0299)	(0.1073)	(0.2942)	(0.0155)	(0.0959)
Hiprior	0.2932	0.9735	0.7702	0.3062	1.0029	0.7951
	(0.2607)	(0.0529)	(0.1659)	(0.2660)	(0.0334)	(0.1543)

Table 11: **Unconditional Euler tests for IND**

	Euler Bond Conditions			Euler Stock Conditions		
Portfolio	$\overline{E}_n^B$	$\overline{E}_b^B$	$\overline{E}_s^B$	$\overline{E}_n^S$	$\overline{E}_b^S$	$\overline{E}_s^S$
NoDur	0.5062	1.0103	0.8904	0.5272	1.0388	0.9171
	(0.2759)	(0.0250)	(0.0919)	(0.2761)	(0.0074)	(0.0779)
Durbl	0.3161	0.9532	0.7250	0.3264	0.9811	0.7460
	(0.3195)	(0.0841)	(0.2352)	(0.3247)	(0.0611)	(0.2242)
Manuf	0.4658	0.9960	0.8452	0.4801	1.0199	0.8665
	(0.3508)	(0.0450)	(0.1522)	(0.3529)	(0.0256)	(0.1386)
Enrgy	0.3593	0.9913	0.8244	0.3748	1.0164	0.8475
	(0.2627)	(0.0374)	(0.1270)	(0.2663)	(0.0219)	(0.1159)
HiTec	0.2879	0.9389	0.6918	0.2971	0.9646	0.7105
	(0.3074)	(0.0974)	(0.2562)	(0.3129)	(0.0763)	(0.2473)
Telcm	0.3336	0.9938	0.8302	0.3500	1.0174	0.8527
	(0.2123)	(0.0285)	(0.0994)	(0.2156)	(0.0169)	(0.0906)
Shops	0.4687	0.9954	0.8436	0.4829	1.0201	0.8654
	(0.3568)	(0.0463)	(0.1561)	(0.3589)	(0.0259)	(0.1419)
Hlth	0.3716	1.0010	0.8546	0.3908	1.0264	0.8790
	(0.1943)	(0.0217)	(0.0783)	(0.1966)	(0.0108)	(0.0697)
Utils	0.5116	1.0160	0.9091	0.5341	1.0374	0.9306
	(0.1576)	(0.0117)	(0.0447)	(0.1572)	(0.0045)	(0.0384)
Other	0.3565	0.9845	0.8053	0.3703	1.0101	0.8279
	(0.2913)	(0.0474)	(0.1549)	(0.2954)	(0.0295)	(0.1431)

12 Deriving the participation cost

Consider at date t a household i of type b, that is, with $\tilde{h}_t^i = 1$ and $h_t^i = 0$. Its intertemporal utility $V_t^{b,i}$ is:

$$\begin{aligned} V_t^{b,i} &= \frac{\left(c_t^{b,i}\right)^{1-\gamma_b}}{1-\gamma_b} + \beta_b \mathbb{E}_t \left[\frac{\left(c_{t+1}^{b,i}\right)^{1-\gamma_b}}{1-\gamma_b} \right] + \beta_b^2 \mathbb{E}_t \left[V_{t+2}^{b,i} \right], \\ &= \frac{\left(c_t^{b,i}\right)^{1-\gamma_b}}{1-\gamma_b} \left(1 + \beta_b \mathbb{E}_t e^{(1-\gamma_b)\Delta \log c_{t+1}^{b,i}} \right) + \beta_b^2 \mathbb{E}_t \left[V_{t+2}^{b,i} \right]. \end{aligned} \tag{12.1}$$

We consider the following thought experiment. Household i is constrained to participate in the stock market at date t. Household i remains endowed with individual preferences but must switch to the consumption growth process of stockholders, $\Delta \log c_{t+1}^{i,s}$. To compensate household i for this constrained stock market participation, it receives a flat amount $\tau_t^{b,i}$ at date t. We assume that this amount is fully consumed at date t. The intertemporal utility of this constrained household is denoted by $V_t^{b,i,s}$ and can be expressed as follows:

$$V_t^{b,i,s} = \frac{\left(c_t^{b,i} + \tau_t^{b,i}\right)^{1-\gamma_b}}{1-\gamma_b} \left(1 + \beta_b \mathbb{E}_t e^{(1-\gamma_b)\Delta \log c_{t+1}^{s,i}}\right) + \mathbb{E}_t \left[\beta^2 V_{t+2}^{b,i,s}\right]. \tag{12.2}$$

The compensation τ_t^i exactly offsets the forced participation if $V_t^{b,i,s} = V_t^{b,i}$. If we assume that the constrained participation has no effect after date $t+2$, the equality $V_t^{b,i,s} = V_t^{b,i}$ can be simplified using equations (12.1) and (12.2) as follows

$$\frac{\left(c_t^{b,i} + \tau_t^{b,i}\right)^{1-\gamma_b}}{1-\gamma_b} \left(1 + \beta_b \mathbb{E}_t e^{(1-\gamma_b)\Delta \log c_{t+1}^{s,i}}\right) = \frac{\left(c_t^{b,i}\right)^{1-\gamma_b}}{1-\gamma_b} \left(1 + \beta_b \mathbb{E}_t e^{(1-\gamma_b)\Delta \log c_{t+1}^{b,i}}\right),$$

or:

$$\frac{\tau_t^{b,i}}{c_t^{b,i}} = \frac{\left(1 + \beta_b \mathbb{E}_t e^{(1-\gamma_b)\Delta \log c_{t+1}^{b,i}}\right)^{\frac{1}{1-\gamma_b}}}{\left(1 + \beta_b \mathbb{E}_t e^{(1-\gamma_b)\Delta \log c_{t+1}^{s,i}}\right)^{\frac{1}{1-\gamma_b}}} - 1. \tag{12.3}$$

We will compute conditional expectation by conditioning on ω_{t+1} (more precisely, the filtration generated by (ω_t)). Formally:

$$\mathbb{E}_t \left[e^{(1-\gamma_b)\Delta \log c_{t+1}^{b,i}}\right] = \mathbb{E}_t \left[\mathbb{E}\left[e^{(1-\gamma_b)\Delta \log c_{t+1}^{b,i}} | \omega_{t+1}\right]\right].$$

Using the dynamics of the log-consumption growth, we have:

$$\begin{aligned}
(1-\gamma_b)\Delta \log c_{t+1}^{b,i} &= \frac{1-\gamma_b}{\gamma_b}\left\{\log \beta_b + \log(R_{t+1}^f) + \kappa_b \omega_{t+1} + \sigma_b \varepsilon_{t+1}^i + \psi_b \sqrt{\omega_{t+1}} u_{t+1}\right\}, \\
&\sim_{\omega_{t+1}} \mathcal{N}\left(\frac{1-\gamma_b}{\gamma_b}\left(\log \beta_b + \log(R_{t+1}^f) + \kappa_b \omega_{t+1}\right), \left(\frac{1-\gamma_b}{\gamma_b}\right)^2 \left(\sigma_b^2 + \psi_b^2 \omega_{t+1}\right)\right), \\
(1-\gamma_b)\Delta \log c_{t+1}^{s,i} &= \frac{1-\gamma_b}{\gamma_s}\left\{\log \beta_s + \log(R_{t+1}^f) + \kappa_s \omega_{t+1} + \sigma_s \varepsilon_{t+1}^i + \psi_s \sqrt{\omega_{t+1}} u_{t+1}\right\}, \\
&\sim_{\omega_{t+1}} \mathcal{N}\left(\frac{1-\gamma_b}{\gamma_s}\left(\log \beta_s + \log(R_{t+1}^f) + \kappa_s \omega_{t+1}\right), \left(\frac{1-\gamma_b}{\gamma_s}\right)^2 \left(\sigma_s^2 + \psi_s^2 \omega_{t+1}\right)\right).
\end{aligned}$$

In the above equations, $\sim_{\omega_{t+1}}$ denotes the law conditional on ω_{t+1}. We deduce:

$$\mathbb{E}_t\left[e^{(1-\gamma_b)\Delta\log c_{t+1}^{b,i}}|\omega_{t+1}\right] = \\ \exp\left(\frac{1-\gamma_b}{\gamma_b}\left(\log\beta_b+\log(R_{t+1}^f)+\kappa_b\omega_{t+1}\right)+\frac{1}{2}\left(\frac{1-\gamma_b}{\gamma_b}\right)^2\left(\sigma_b^2+\psi_b^2\omega_{t+1}\right)\right),$$

and

$$\mathbb{E}_t\left[e^{(1-\gamma_b)\Delta\log c_{t+1}^{s,i}}|\omega_{t+1}\right] = \\ \exp\left(\frac{1-\gamma_b}{\gamma_s}\left(\log\beta_s+\log(R_{t+1}^f)+\kappa_s\omega_{t+1}\right)+\frac{1}{2}\left(\frac{1-\gamma_b}{\gamma_s}\right)^2\left(\sigma_s^2+\psi_s^2\omega_{t+1}\right)\right).$$

We deduce:

$$\beta_b\mathbb{E}_t[e^{(1-\gamma_b)\Delta\log c_{t+1}^{b,i}}] = \beta_b^{\frac{1}{\gamma_b}}\left(R_{t+1}^f\right)^{\frac{1-\gamma_b}{\gamma_b}} e^{\left(\frac{1-\gamma_b}{\gamma_b}\right)^2\frac{\sigma_b^2}{2}}\mathbb{E}\left[e^{\left(\kappa_b+\frac{1-\gamma_b}{\gamma_b}\frac{\psi_b^2}{2}\right)\frac{1-\gamma_b}{\gamma_b}\omega_{t+1}}\right],$$
$$\beta_b\mathbb{E}_t[e^{(1-\gamma_b)\Delta\log c_{t+1}^{s,i}}] = \beta_b\beta_s^{\frac{1-\gamma_b}{\gamma_s}}\left(R_{t+1}^f\right)^{\frac{1-\gamma_b}{\gamma_s}} e^{\left(\frac{1-\gamma_b}{\gamma_s}\right)^2\frac{\sigma_s^2}{2}}\mathbb{E}\left[e^{\left(\kappa_s+\frac{1-\gamma_b}{\gamma_s}\frac{\psi_s^2}{2}\right)\frac{1-\gamma_b}{\gamma_s}\omega_{t+1}}\right].$$

Finally:

$$\frac{\tau_t^{b,i}}{c_t^{b,i}} = \frac{\left(1+\beta_b^{\frac{1}{\gamma_b}}\left(R_{t+1}^f\right)^{\frac{1-\gamma_b}{\gamma_b}} e^{\left(\frac{1-\gamma_b}{\gamma_b}\right)^2\frac{\sigma_b^2}{2}}\mathbb{E}e^{\left(\kappa_b+\frac{1-\gamma_b}{\gamma_b}\frac{\psi_b^2}{2}\right)\frac{1-\gamma_b}{\gamma_b}\omega_{t+1}}\right)^{\frac{1}{1-\gamma_b}}}{\left(1+\beta_b\beta_s^{\frac{1-\gamma_b}{\gamma_s}}\left(R_{t+1}^f\right)^{\frac{1-\gamma_b}{\gamma_s}} e^{\left(\frac{1-\gamma_b}{\gamma_s}\right)^2\frac{\sigma_s^2}{2}}\mathbb{E}e^{\left(\kappa_s+\frac{1-\gamma_b}{\gamma_s}\frac{\psi_s^2}{2}\right)\frac{1-\gamma_b}{\gamma_s}\omega_{t+1}}\right)^{\frac{1}{1-\gamma_b}}} - 1. \tag{12.4}$$

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